Description Logics for the Representation of Aggregated Objects

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Abstract. Aggregated objects play an important role in many knowledge representation applications. For the adequate representation of aggregated objects, it is crucial to represent part-whole relations. We discuss properties of part-whole relations and extend the description logic $\mathcal{ALC}$ with means for the adequate representation of part-whole relations and thus of aggregated objects.

1 Motivation

Description logics are a family of knowledge representation formalisms well-suited for the representation of and reasoning about configurations [27, 21], ontologies [19], and database schemata, where they can support schema design, evolution, and query optimisation [4, 7], source integration in heterogeneous databases/data warehouses [5, 6], and conceptual modeling of multidimensional aggregation [11].

In all these applications, aggregated objects play a central role, that is, objects that are composed of various parts, which again can be composite, etc. It is natural to describe an aggregated object by means of its parts and vice versa, to describe parts by means of the aggregate they belong to. For example, the following statements describe a control rod and a reactor core by means of their parts and wholes, where $\sqsubseteq$ is a subsumption (implication) relationship:

\[
\begin{align*}
\text{Control-rod} & \sqsubseteq \text{Device} \sqcap \exists \text{part-of. Reactor-core} \\
\text{Reactor-core} & \sqsubseteq \text{Device} \sqcap \exists \text{has-part. Control-rod} \sqcap \exists \text{part-of. Nuclear-reactor}
\end{align*}
\]

Referring to wholes a part belongs to, we use the part-whole relation (written $\text{part-of}$ and abbreviated pw-relation). Vice versa, to refer to the parts of an object, we use the has-part relation, which is the inverse of the pw-relation, is written has-part, and abbreviated hp-relation. It is commonly believed [1] that only a formalism with very high expressive power can represent pw-relations and aggregated objects adequately. In this paper, we argue in how far the high expressive power of the description logic $\mathcal{SHIQ}$ is crucial for the adequate representation of aggregated objects. Despite $\mathcal{SHIQ}$’s high expressiveness, there is a practicable reasoning algorithm which decides inference problems such as satisfiability and subsumption of $\mathcal{SHIQ}$ concepts w.r.t. to (possibly cyclic) terminological knowledge bases.

2 Some properties of Part-Whole Relations

In contrast to, for example, the relation likes, the pw-relation has a variety of properties. For a complete collection of these properties, we refer to [25]. Most importantly, the general pw-relation is a strict partial order, i.e., it is transitive and asymmetric (and hence irreflexive). That is, if $x$ part-of $y$ and $y$ part-of $z$, then $x$ part-of $z$, and if $x$ part-of $y$, then not $y$ part-of $x$. Moreover, an aggregated object has at least two parts where none is a part of the other. Next, we might consider to assume that two objects consisting of the same parts are identical. As a last example, we might assume the existence of atoms, i.e., indivisible objects of which all other objects are composed. This is equivalent to assume that has-part is well-founded and thus to exclude infinite chains $x_0 \text{ has-part } x_1 \text{ has-part } x_2 \ldots$

Sub-Relations of the General Part-Whole Relation Besides the properties mentioned above, the pw-relation is assumed to have various sub-relations, like, for example, the relation between a component and its composite (e.g. between a motor and the car the motor is in), the relation between stuff and an object containing this stuff (e.g. between metal and a car), or the relation between a member and a collection it belongs to (e.g. between a tree and the forest this tree belongs to).

These pw-relations are subject of several investigations and discussions; see, for example, [28, 12, 18]. However, various questions concerning pw-relations are still open. Let part-of denote sub-relations of the general pw-relation:

- How are pw-relations defined, i.e., what is a necessary and sufficient condition $\Phi_i(x, y)$ for a part $x$ of $y$ to be in the relation part-of, with $y$, i.e., which $\Phi_i$ satisfies $x$ part-of $y \Leftrightarrow (\Phi_i(x, y) \land x$ part-of $y)$?

- What is the interrelationship between pw-relations? Is one a specialisation of the other, or are they all independent?

- How do they interact with each other and with the general pw-relation? I.e., for which $i, j, k$ does the following implication hold:

\[
(x \text{ part-of } i y \land y \text{ part-of } j z) \Rightarrow x \text{ part-of } k z?
\]

- What is a complete collection of pw-relations? And which of these relations are of importance in a specific application?

To help answering some of these questions, we present a scheme to structure pw-relations in Figure 2. At the top, you find the general pw-relation part-of. The only property we impose on part-of is that it is a strict partial ordering. Then part-of is specialised along three dimensions².

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² The specialisations are not assumed to be disjoint.
Mereological collections refers to the pw-relation in the rather strict sense of classical mereology [25]. Besides being a strict partial order, mereological collections must satisfy a variety of properties. These properties determine the structures one accepts as models and those one wants to exclude from being models. For example, the supplementation principle says that if a whole has a strict part, then it also has another part that is independent from the first one. Another condition, atomicity, excludes structures with infinite ascending chains from being models.

Integral pw-relations are sub-relations of the general part-whole relation which involve certain integrity conditions on the parts. For $x$ to be a part of $y$ with respect to an integral pw-relation, $x$ has to be a part of $y$, and $x$ has to satisfy the integrity condition associated with this relation. Integral pw-relations are specialised in a natural way by specialising the associated integrity conditions.

For example, for the substantial-integral pw-relation to hold between $x$ and $y$, $x$ must be a part of $y$, and $x$ must be integral with respect to the substantial aspect, i.e., $x$ must consist completely of some substance, which is, since $x$ is also a part of $y$, also present in $y$. Another example to mention is the geometric-integral pw-relation, which holds between $x$ and $y$ if $x$ is a part of $y$ and $x$ is geometrically integral, for example, a convex body. It is important to note that all these integrity conditions are imposed on the parts only, independent of the whole.

Composed pw-relations are characterised by an additional relation which has to hold between a part and its whole. For example, the component-aggregate relation holds between $x$ and $y$ if $x$ is a part of $y$, $x$ is an integral object (with respect to a certain integrity condition) and $x$ is functional for $y$, i.e., the functioning of $y$ depends on $x$. They are specialised by specialising the associated additional relations.

This additional relation (such as “being functional for” in the previous example) is the reason why composed pw-relations are, in general, not transitive: For a composed pw-relation to be transitive, the additional relation must be transitive.

This way of structuring pw-relations has two advantages: First of all, the interaction between pw-relations is given through their definition and the definition of the integrity conditions and additional relations. For example, the following implications are immediate consequences of the above definitions, where $\text{int}_r$ are integrity conditions associated with integral pw-relations $\text{int}^r\text{-part-of}$, and $\text{comp}_i$ are additional relations for composed pw-relations $\text{comp}^i\text{-part-of}$.

\[
\begin{align*}
    x \text{ integral-part-of } y \land y \text{ part-of } z & \Rightarrow x \text{ integral-part-of } z \\
    x \text{ int}^1\text{-part-of } y \land (\forall z. \text{int}_1(z) \Rightarrow \text{int}_2(z)) & \Rightarrow x \text{ int}^2\text{-part-of } y \\
    x \text{ comp}^1\text{-part-of } y \land (\forall z_1,z_2. \text{comp}_1(z_1,z_2) \Rightarrow \text{comp}_2(z_1,z_2)) & \Rightarrow x \text{ comp}^2\text{-part-of } y
\end{align*}
\]

Secondly, the pw-relations identified in the literature can be placed in our taxonomy. For example, the classification of pw-relations from [28] into our taxonomy is given in Table 1.

3 Introduction to Description Logics

We briefly introduce syntax and semantics of the well-known, basic description logic $\mathcal{ALC}$ [24].

Definition 1 Let $C$ be a set of concept names and let $R$ be a set of role names. The set of $\mathcal{ALC}$-concepts is the smallest set such that

1. every concept name is a concept and
2. if $C$ and $D$ are concepts and $R$ is a role name, then $(C \sqcap D), (C \cup D), (\neg C), (\forall R.C), (\exists R.C)$ are concepts.

The semantics is given by an interpretation $I = (\Delta^I, \cdot^I)$, which consists of a set $\Delta^I$, called the domain of $I$, and a function $\cdot^I$ which maps every concept to a subset of $\Delta^I$ and every role to a subset of

Figure 1. A Taxonomy of Part-Whole Relations.
\[ \Delta^2 \times \Delta^2 \text{ such that} \]

\[
(C \cap D)^2 = C^2 \cap D^2, \\
(C \cup D)^2 = C^2 \cup D^2, \\
C^2 = C^2 \setminus C^2, \\
(\exists R.C)^2 = \{d \in \Delta^2 \mid \text{There exists an } e \in \Delta^2 \text{ with } (d, e) \in R^2 \text{ and } e \in C^2\}, \\
(\forall R.C)^2 = \{d \in \Delta^2 \mid \text{For all } e \in \Delta^2, (d, e) \in R^2, \text{ then } e \in C^2\}.
\]

A concept \( C \) is called satisfiable iff there is some interpretation \( I \) such that \( C^2 \neq \emptyset \). Such an interpretation is called a model of \( C \). A concept \( D \) subsumes a concept \( C \) (written \( C \sqsubseteq D \)) iff \( C^2 \subseteq D^2 \) holds for each interpretation \( I \).

So far, \( \mathcal{ALC} \) allows us to describe concepts relevant in an application domain. For example, the following concept describes a cooled mixer-reactor:

\[
\text{Mat-Object} \sqcap (\exists \text{has-part}. \text{Cooler}) \sqcap (\exists \text{has-part}. \text{Mixer}) \sqcap (\forall \text{contains}. (\text{Hom-phase} \sqcup \text{Inhom-phase})
\]

In [24], it was shown that reasoning in \( \mathcal{ALC} \) (i.e., subsumption and satisfiability of \( \mathcal{ALC} \)-concepts) is decidable, more precisely, it is PSPACE-complete. Although this is far more complex than what is commonly assumed to be tractable, it turned out that the corresponding algorithms are amenable to optimisation and behave quite well in practice [3, 14, 13].

The terminological knowledge of an application domain can be fixed in a so-called terminology.

**Definition 2** A terminological axiom is an expression of the form \( C \sqsubseteq D \), where \( C \) and \( D \) are concepts. A terminology is a finite set of terminological axioms. We use \( C \equiv D \) as abbreviation for \( C \sqsubseteq D \) and \( D \sqsubseteq C \).

An interpretation \( I \) satisfies a terminological axiom \( C \sqsubseteq D \) iff \( C^2 \subseteq D^2 \), and it satisfies a terminology \( T \) iff it satisfies each axiom in it. Such an interpretation is called a model of \( T \). A concept \( C \) is subsumed by a concept \( D \) with respect to a terminology \( T \) iff \( C^2 \subseteq D^2 \) holds for each model \( I \) of \( T \). A concept \( C \) is satisfiable with respect to a terminology \( T \) iff \( C^2 \neq \emptyset \) holds for some model \( I \) of \( T \).

For example, in Section 1, you find two terminological axioms, one describing a control rod, the other describing a reactor core. Please note that this terminology contains a cycle: the description of control rod refers to reactor core, whose description refers to control rod. Besides the underlying description logic, knowledge representation systems based on description logics also differ in whether they allow for those cyclic terminologies or whether they restrict the left hand side of terminologies to concept names and disallow cycles.

## 4 Description Logics for Part-Whole Relations

In this section, we gradually extend \( \mathcal{ALC} \) to give it more of the expressive power required for the representation of aggregated objects.

**Transitivity of part-of** One shortcoming of \( \mathcal{ALC} \), when used for the representation of aggregated objects, is that it does not provide any means for the representation of transitive relations. For example, in \( \mathcal{ALC} \), the concept

\[
\text{Device} \sqcap \exists \text{has-part.} (\text{Reactor-core} \sqcap \exists \text{has-part}. \text{Control-rod})
\]

is not subsumed by

\[
\text{Device} \sqcap \exists \text{has-part}. \text{Control-rod},
\]

although the first concept is a specialisation of the second one under the assumption that \( \text{has-part} \) is interpreted as a transitive relation.

There are basically three possibilities to overcome this shortcoming: We extend \( \mathcal{ALC} \) with

1. the transitive closure of roles [2]. That is, for a role name \( R \), we allow the use of its transitive closure \( R^+ \) in concepts of the form \( \exists R. C \) and \( \forall R^+ . C \), and define interpretations to interpret \( R^+ \) as the transitive closure of \( R^2 \).

Unfortunately, this extension leads to \text{ExPTIME}-completeness of reasoning [22]—even with respect to empty terminologies.

2. the transitive orbit of roles [20], whose syntax is defined analogously to the one of the transitive closure. An interpretation is then defined to interpret the transitive orbit \( R^+ \) of a role name \( R \) as some transitive relation containing \( R^2 \). Although this seems to be much weaker an extension than the one by the transitive closure, it has the same consequences for the computational complexity, i.e., reasoning in \( \mathcal{ALC} \) with transitive orbits is \text{ExPTIME}-complete [20].

3. transitive roles, i.e., we allow the user to specify a subset \( R_+ \subseteq R \) of transitive role names, and define interpretations to interpret transitive role names \( S \in R_+ \) as transitive relations. Reasoning in \( \mathcal{ALC} \) with transitive roles could be shown to be in the same complexity class as pure \( \mathcal{ALC} \), namely \text{PSPACE} [20].

The way of decomposing aggregated objects strongly depends on the individual taste, aims, and circumstances. Hence modeling aggregated objects using a direct hp-relation \( \text{has-d-part} \) is problematic. For example, defining \( \text{Human} \sqsubseteq \exists \text{has-d-part}. \text{Abdomen} \) is as sensible as defining \( \text{Human} \sqsubseteq \exists \text{has-d-part}. \text{Stomach} \) and \( \text{Stomach} \sqsubseteq \exists \text{has-d-part}. \text{Abdomen} \). However, this yields models where a human has an abdomen as a direct part, and where the abdomen is also a part of the human’s stomach—which clearly
clashes with our intuition of direct pw-relations. Hence we believe
that the third and “cheapest” extension is sufficient for most appli-
cations. By $S$, we refer to the description logic $ALC$ extended with
transitive roles.\footnote{The logic $S$ has previously been called $ALC_{R^+}$, but this becomes too
cumbersome when adding letters to represent additional features.}

Obviously, $S$ provides the means to represent the general pw-
relation as a transitive relation by asserting $\text{part-of} \in R_+$. Addi-
tionally, since $S$ has the tree-model property, for each model $I$, we
can construct a model $I'$ in which $\text{part-of}^R$ is a strict partial or-
dering. Hence in $S$ and all its extensions, we can model the general
pw-relation. Moreover, we can also represent integral pw-relations:

Let $C_{\text{int}}$ be a concept describing a certain integrity condi-
tion, replace $\exists \text{int-part-of}.C$ by $C_{\text{int}} \sqsubset \exists \text{part-of}.C$ and
replace $\forall \text{int-part-of}.C$ by $\neg C_{\text{int}} \sqcup \forall \text{part-of}.C$.

Obviously, this substitution yields a concept in which $\text{int-part-of}$ is interpreted according to the intended mean-
ing, i.e., each instance of $\exists \text{int-part-of}.C$ is integral w.r.t. to the
condition $C_{\text{int}}$. Please note that different kinds integrity conditions
require different expressive power—possibly more than $S$ provides.

**Either part-of or has-part?** When describing concepts of an
application domain using $S$ and using the pw- as well as the hp-
relation, we risk that the description is not adequate: $\text{part-of}$ is
the inverse of $\text{has-part}$ (and vice versa), a fact that cannot be ex-
pressed in $S$. For example, extending the terminology in Section 1
with

\begin{equation}
\text{Nuclear reactor} \sqcap \exists \text{has-part}.\text{Faulty} \sqsubseteq \text{Dangerous},
\end{equation}

we would assume that Control Rod $\sqcap \exists \text{has-part}.\text{Faulty}$ is subsumed by
$\exists \text{part-of}.\text{Dangerous}$ w.r.t. to this terminology—which is only
the case if $\text{part-of}$ were the inverse of $\text{has-part}$. Hence in $S$,
we must decide whether (1) we use either $\text{part-of}$ or $\text{has-part}$,
(2) we use $\text{part-of}$ and $\text{has-part}$ and live with the fact that
our model is inadequate in the sense of the previous example, or (3)
extend $S$ with inverse roles. We have decided to choose option 3:

**Definition 3** The description logic $SI$ is obtained from $S$ by allow-
ing, additionally, for inverse role names $R^\leftrightarrow$ with $R \in R$ to occur in
the place of role names. An interpretation must satisfy, additionally,

\begin{equation}
(R^\leftrightarrow)^I := \{ (e, d) \mid (d, e) \in R^I \}.
\end{equation}

Hence in $SI$, we can describe both objects by means of the
wholes they belong to and by means of the parts they have. Substituting $\text{has-part}$ for $\text{part-of}$ in the last example
yields that Control Rod $\sqcap \exists \text{part-of}.\text{Faulty}$ is indeed subsumed by
$\exists \text{part-of}.\text{Dangerous}$.

Fortunately, it could be shown that reasoning in $SI$ is still
PSPACE-complete [16].

**Composed Sub-Part-Whole Relations** To additionally represent
composed pw-relations, we extend $SI$ with role-hierarchies, which
allow the user to represent composed part-whole relations as sub-
roles of the general pw-relation.

**Definition 4** A role inclusion axiom is an expression of the form
$R \sqsubseteq S$, where $R$ and $S$ are (possibly inverse) roles. A role hierarchy
is a finite set of role inclusion axioms. An interpretation $I$ satisfies
a role hierarchy $R$ iff $R^I \subseteq S^I$ for each $R \sqsubseteq S$ in $I$. Such an
interpretation is called a model of $R$.

\begin{align*}
\text{SI} & \text{ is the extension of SI with role hierarchies.} \\
\text{For a role hierarchy } R \text{, the sub-role relation is the transitive closure}
& \text{ of } R \sqsubseteq R \cup \{ R^\leftrightarrow \sqsubseteq S^\leftrightarrow \mid R \sqsubseteq S \in R \}. \footnote{We assume that $R^\leftrightarrow = R$.}
\end{align*}

Adding role hierarchies to $SI$ has mainly two consequences: First,
we can specify composed pw-relations, i.e., we can introduce (pos-
sibly transitive—depending on the additional relation) role names $\text{comp-i-part-of}$ and add role inclusion axioms

\begin{align*}
\text{comp-i-part-of} & \sqsubseteq \text{part-of or} \\
\text{comp-i-part-of} & \sqsubseteq \text{comp-j-part-of}.
\end{align*}

However, we cannot define (in the sense of axiomatise) these addi-
tional relations. For example, we cannot specify what conditions a
part and a whole must satisfy for the component-aggregate relation
to hold between them.

Second, $SHI$ (as well as $SH$ and $SHIQ$) has the expressive
power for the internalisation of terminologies [2, 15]. This technique
polynomially reduces reasoning w.r.t. a (possibly cyclic) terminol-
ogy to pure concept reasoning. First, we introduce a new transitive
role name $U \in R_+$ and specify that $U$ is a super-role of all roles
and their respective inverses. Then, a concept $C$ is satisfiable w.r.t.
$\{ C_i \sqsubseteq D_i \mid 1 \leq i \leq n \}$ iff $C \sqcap \bigcap_{1 \leq i \leq n} \neg C_i \sqcup D_i$ is satisfiable.

**Number Restrictions** In general, when modeling an application
domain, it seems to be natural to describe an object by restricting
the number of objects it is related to via a certain relation. For ex-
ample, the first of the following concepts describes pipes as those
connections having exactly 1 input and 1 output, whereas the second
concept describes forks as those connections having 1 input and at
least 2 outputs:

\begin{align*}
\text{Connection} & \{(= 1 \text{ c-part-of}^- \text{ In}) \sqcap (= 1 \text{ c-part-of}^- \text{ Out}) \} \\
\text{Connection} & \{(= 1 \text{ c-part-of}^- \text{ In}) \sqcap (\geq 2 \text{ c-part-of}^- \text{ Out}) \}
\end{align*}

Since number restrictions are mostly “harmless” from an algorithmic
point of view [26], we have added them to $SHI$.

**Definition 5** A (possibly inverse) role is called simple if it is neither
transitive nor has a transitive sub-role.

$SHIQ$ is obtained from $SHI$ by allowing, additionally, for con-
cepts of the form $(\geq n.R.C)$ and $(\leq n.R.C)$ for $n$ a non-negative
integer, $R$ a simple role, and $C$ a $SHIQ$-concept.

\begin{align*}
(\geq n.R.C)^I & = \{ x \mid \exists y (y, x, y) \in R^I \text{ and } y \in C^I \geq n \} \\
(\leq n.R.C)^I & = \{ x \mid \exists y (y, x, y) \in R^I \text{ and } y \in C^I \leq n \}.
\end{align*}

**5 Discussion**

**Computational Properties of $SHIQ$:** It is known that reasoning
for the highly expressive description logic $CIT^Q$ is EXPTIME-complete
[9]. Now $SHIQ$ is less expressive than $CITQ$ and in the same compu-
tability class, namely EXPTIME-complete (this is a consequence of

\begin{align*}
\text{5 Discussion} & \text{ Computational Properties of } SHIQ: \text{ It is known that reasoning for}
& \text{the highly expressive description logic } CIT^Q \text{ is EXPTIME-complete}
& \text{[9]. Now } SHIQ \text{ is less expressive than } CITQ \text{ and in the same com-}
& \text{plexity class, namely EXPTIME-complete (this is a consequence of}
\end{align*}

\begin{align*}
\text{We assume that } R^\leftrightarrow = R. \\
\text{We use } \{ = 1 \text{ } R \} \text{ as a short hand for } \{ \leq 1 \text{ } R \} \sqcap \{ \geq 1 \text{ } R \}
\end{align*}

\begin{align*}
\text{Basically, } CITQ \text{ is obtained from } SHIQ \text{ by allowing for regular expres-
sions of roles in concepts of the form } \exists R.C \text{ or } \forall R.C. \text{ The main difference}
& \text{between } CITQ \text{ and } SHIQ \text{ is that the latter has the transitive closure op-
erator on roles, whereas } SHIQ \text{ only has transitive roles and role hierarchies.}
\end{align*}
the fact that SHIQ is a fragment of CIQ, which is in EXPTIME, and the fact that SHIQ is an extension of ALC with transitive orbits, which is EXPTIME-hard [20]—which is basically due to the fact that we can introduce a transitive super-role of all other roles. Hence a question naturally arising here is why we should be interested in SHIQ. The answer is that there is a direct tableau-based decision procedure for the satisfiability and subsumption of SHIQ-concepts with respect to role hierarchies and possibly cyclic terminologies [16]. After first experiments with the extension iFaCT of the description logic system FaCT [14], we believe that this algorithm is as practicable and well-behaved in practice as the one implemented in FaCT. One reason for this good behaviour could be that a large fragment of SHIQ, namely the one obtained by omitting role hierarchies and thus the ability to internalise terminologies is PSPACE-complete [16]. Another nice property of the tableau-based algorithm for SHIQ is that it does not have an equivalent to the analytic cut rule [10], a rule that introduces a large amount of nondeterminism and that is used in satisfiability algorithms in the presence of the transitive closure operator and inverse roles.

**Expressive power of SHIQ**: In SHIQ and CIQ, we can internalise terminologies, hence polynomially reduce reasoning w.r.t. terminologies to pure concept reasoning.

Like CIQ, SHIQ lacks the finite model property (i.e., some concepts only have infinite models), and has the tree model property (i.e., each satisfiable concept has a tree model).

If we assume the existence of atoms, the SHIQ reasoning algorithm in [16] is not yet satisfactory since it admits models where—even though part-of is interpreted as a strict partial ordering—part-of is not well-founded. Since part-of should be interpreted as an asymmetric relation, having a reasoning algorithm for finite models would neither be satisfactory. Instead, atomicity requires a reasoning algorithm that decides the existence of a model where part-of is a well-founded, strict partial ordering [8].

Finally, since SHIQ has the tree model property, we cannot model what is called “inheritance along pw-relations”. For example, we cannot model that the owner of the reactor is also the owner of the mixer and the cooler, i.e., that the property owner is “inherited” along the hp-relation in [1], some approaches are discussed that have this kind of expressive power. Using role value maps (i.e., concepts like has-component owner ⊆ owner) to model this “inheritance” is not a good idea: even for very weak description logics, the extension with role-value maps leads to the undecidability of subsumption—and thus the ability to internalise terminologies—is PSPACE-complete [16]. Another nice property of the tableau-based algorithm for SHIQ is that it does not have an equivalent to the analytic cut rule [10], a rule that introduces a large amount of nondeterminism and that is used in satisfiability algorithms in the presence of the transitive closure operator and inverse roles.

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