Encoding information fusion in possibilistic logic: A general framework for rational syntactic merging

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Abstract. The problem of merging multiple sources information is central in many information processing areas such as databases integration problems, multiple criteria decision making, expert opinion pooling, etc. Recently, several approaches have been proposed to merge classical propositional bases, or sets of (non-prioritized) goals. These approaches are in general semantically defined. Like in belief revision, they use priorities, generally based on Dalal's distance, for merging the classical bases and return a new classical base as a result. An immediate consequence of the generation of a classical base is the impossibility of iterating the fusion process in a coherent way w.r.t. priorities since the underlying ordering is lost. This paper presents a general approach for fusing prioritized bases, both semantically and syntactically, when priorities are represented in the possibilistic logic framework. We show that the approaches which have been recently proposed for merging classical propositional bases can be embedded in this setting. The result is then a prioritized base, and hence the process can be coherently iterated. Moreover, we also provide a syntactic counterpart for the fusion of classical bases.

1 Introduction

It is well known that priorities are very important in belief revision or for fusing multiple source information. Gärdenfors [5] has proved that any revision process which satisfies the so-called AGM postulates is based on some implicit ordering. Priorities are crucial as well to deal with conflicting sources. Even when the information provided by the sources take the form of classical bases, (which represent non-stratified sets of pieces of knowledge or of goals without explicit priorities), several authors, e.g. Lin [9], Konieczny and Pino Pérez [6, 7], Lin and Mendelzon [10, 11], Liberatore and Schaerf [8] and Revesz [12, 13], extract implicit orderings from these classical bases. For instance, consider the following example [12] where a teacher asks students which among the following languages SQL (denoted by s), O_2 (denoted by o) and $Datalog$ (denoted by d) they would like to learn. If one provides the classical propositional base $(s \vee o) \wedge \neg d$, then the teacher assumes that the student implicitly gives two independent sub-goals: "learning either SQL or O_2, or both" and "not learning $Datalog$". This means that the student implicitly gives a set of prioritized goals where the situations $(s \wedge o \wedge \neg d)$, $(\neg s \wedge o \wedge \neg d)$ and $(s \wedge \neg o \wedge \neg d)$ are the preferred ones (since they satisfy the two sub-goals "$s \vee o$" and "$\neg d$"), that the situations $\neg s \wedge \neg o \wedge \neg d$, $s \wedge o \wedge d$, $s \wedge \neg o \wedge d$, $\neg s \wedge o \wedge d$ are less preferred (since they only satisfy one sub-goal), and lastly that the situation $\neg s \wedge \neg o \wedge d$ is the least preferred one since both sub-goals are falsified. These priorities can be obtained using Dalal's distance [3].

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In this paper we propose a general approach for fusing prioritized propositional bases in the framework of possibilistic logic. Prioritized propositional bases are sets of weighted formulas of the form $B = \{(\phi_i, a_i) : i = 1, n\}$ where ϕ_i is a classical formula, and a_i belongs to $[0, 1]$ encoding the level of priority of ϕ_i. Each possibilistic base B induces a possibility distribution π_B, which rank-orders the different interpretations. We show that this general framework allows us to recover classical fusion methods recently proposed in [6, 7, 9, 10, 11]. This is obtained by the syntactic association of a set of weighted formulas to each classical base, and by providing the possibilistic counterpart of any classical merging operator. We show that this process can be iterated in a coherent way, and also provides a syntactic way for computing classical fusion operators which are only semantically defined generally.
Technically speaking our aim in this paper is, given a multi-set of classical propositional bases E, and a boolean merging operator Δ, to show how to compute the set of models of the resulting base using possibilistic merging operators. To reach this aim, we first associate to each K_i in E a prioritized base B_{K_i}. Roughly speaking, this base should be such that $\pi_{B_{K_i}}$, the possibility distribution associated to B_{K_i}, be a function of the Dalal distance associated to K_i. Let $\mathcal{B} = \{B_{K_1}, ..., B_{K_n}\}$ be the prioritized counterpart of $E = \{K_1, ..., K_n\}$. Then for each Δ applied to E and yielding the propositional base $\Delta(E)$, we define a possibilistic aggregation operator $\oplus$ applied to $\mathcal{B} = \{B_{K_1}, ..., B_{K_n}\}$ and yielding to a prioritized base $\mathcal{B}_{\oplus}$. This base should be such that the set of preferred models in $\pi_{\mathcal{B}_{\oplus}}$ is equal to $[\Delta(E)]$ the set of models of $\Delta(E)$, as pictured on Figure 1.

$$
\begin{array}{ccc}
E = \{K_1, ..., K_n\} & \longrightarrow & \mathcal{B} = \{B_{K_1}, ..., B_{K_n}\} \\
\downarrow \Delta & & \downarrow \oplus \\
d_{\Delta}(\omega, E) & & \mathcal{B}_{\oplus} \text{ s.t. } \pi_{\mathcal{B}_{\oplus}}(\omega) = f(d_{\Delta}(\omega, E)) \\
\searrow & & \swarrow \\
[\Delta(E)] & = & \{\omega^*, max_{\omega \in \Omega} \pi_{\mathcal{B}_{\oplus}}(\omega) = \pi_{\mathcal{B}_{\oplus}}(\omega^*)\}
\end{array}
$$

Figure 1

Let us first recall the fusion of classical propositional bases before giving a brief overview of merging operators in possibilistic logic, and presenting the results of the paper.

2 Fusion of classical propositional bases

2.1 Basic steps and local distances

We consider a propositional language $\mathcal{L}$ over a finite alphabet $\mathcal{P}$ of atoms. Ω denotes the set of all interpretations. Logical equivalence is denoted by $\equiv$. Classical disjunction and conjunction are respectively represented by $\vee, \wedge$. Let ψ be a formula of $\mathcal{L}$, $[\psi]$ denotes the set of all models of ψ. A literal is an atom or a negation of an atom.

K denotes a classical propositional base. Let $E = \{K_1, ..., K_n\}$ $(n \geq 1)$ be a multi-set of n consistent propositional bases to be merged. E is called an information set. We define a merging operator Δ as a function which associates to each information set E a classical propositional base, denoted by $\Delta(E)$.

The three basic steps followed in [6, 7, 9, 10, 11] for merging semantically an information set E by an operator Δ are:

1- Rank-order the set of interpretations Ω with respect to each propositional base K_i by computing a local distance, denoted $d(\omega, K_i)$, between ω and each K_i.

2- Rank-order the set of interpretations Ω w.r.t. all the propositional bases. This leads to the overall distance denoted $d_\Delta(\omega, E)$. The latter, computed from local distances $d(\omega, K_i)$, defines an ordering relation between the interpretations defined as follows:

$$\omega \leq_E^\Delta \omega' \text{ iff } d_\Delta(\omega, E) \leq d_\Delta(\omega', E).$$

3- Compute the models of $\Delta(E)$, the result of the merging process, whose models are minimal with respect to $\leq_E^\Delta$, namely

$$[\Delta(E)] = min(\Omega, \leq_E^\Delta).$$

In the reviewed works [6, 9, 10, 11], the local distance $d(\omega, K_i)$ is based on Dalal's distance [3]. The distance between an interpretation ω and a propositional base K_i is defined by the least number of atoms on which this interpretation differs from some model of the propositional base. More formally,

$$d(\omega, K_i) = min_{\omega' \in [K_i]} dist(\omega, \omega'),$$

where $dist(\omega, \omega')$ is the number of atoms whose valuations differ in the two interpretations (Hamming distance).

Konieczny and Pino Pérez [7] use any distance which is symmetric and such that $dist(\omega, \omega') = 0$ iff $\omega = \omega'$.

Example Let us extend the example of the introduction with three students. The first student wants to only learn SQL or O_2: $K_1 = (s \vee o) \wedge \neg d$. The second wants to only learn $Datalog$ or O_2 but not both: $K_2 = (\neg s \wedge d \wedge \neg o) \vee (\neg s \wedge \neg d \wedge o)$. The third wants to learn the three languages: $K_3 = (s \wedge d \wedge o)$.

Let $\omega_0 = \neg s \neg d \neg o$, $\omega_1 = \neg s \neg d o$, $\omega_2 = \neg s d \neg o$, $\omega_3 = \neg s d o$, $\omega_4 = s \neg d \neg o$, $\omega_5 = s \neg d o$, $\omega_6 = s d \neg o$, $\omega_7 = s d o$.

Dalal's distances between each interpretation and the bases are:

ω	$d(\omega, K_1)$	$d(\omega, K_2)$	$d(\omega, K_3)$
ω_0	1	1	3
ω_1	0	0	2
ω_2	2	0	2
ω_3	1	1	1
ω_4	0	2	2
ω_5	0	1	1
ω_6	1	1	1
ω_7	1	2	0

Table1

2.2 Aggregating local distances

Once $d(\omega, K_i)$ is defined for each K_i, several methods have been proposed in order to aggregate the local distances $d(\omega, K_i)$ according to whether the bases have the same importance or not. In particular the following operators have been proposed:

- Majority operator [10, 11, 12]: $d_\Sigma(\omega, E) = \Sigma_{i=1}^n d(\omega, K_i)$
- Weighted sum operator [12]: $d_{ws}(\omega, E) = \Sigma_{i=1}^n d(\omega, K_i) * \alpha_i$, where α_i's are integers.
- Max-based egalitarist operator [13]: $d_{max}(\omega, E) = max_{i=1,n} d(\omega, K_i)$
- Lexmax-based (or generalized max) egalitarist operator [6, 7]: This aggregating operator compares vectors of distances. Let $d_j^\omega = d(\omega, K_j)$. Denote $d_{gmax}(\omega, E)$ the result of sorting the vector $(d_1^\omega ... d_n^\omega)$ in a decreasing order. Then, $\omega \in [\Delta(E)]$ if there is no ω' such that $d_{gmax}(\omega', E) <_{Lex} d_{gmax}(\omega, E)$, where $<_{Lex}$ is defined as follows:
  Let $S_1 = (s_1 ... s_n)$ and $S_2 = (s'_1 ... s'_n)$ be two sequences of integers given in a decreasing order. Then, $S_1 <_{Lex} S_2$ if and only if $\exists k \leq n$ such that $s_k < s'_k$ and $\forall i < k : s_i = s'_i$.
  Moreover, $S_1 =_{Lex} S_2$ if $\forall k \leq n, s_k = s'_k$.

Example (continued) Let $\alpha_1 = \alpha_3 = 1$, $\alpha_2 = 3$ for ws operator.

ω	Σ	ws	max	$gmax$
ω_0	5	7	3	$(3,1,1)$
ω_1	**2**	**2**	2	$(2,0,0)$
ω_2	4	4	2	$(2,2,0)$
ω_3	3	5	**1**	$(1,1,1)$
ω_4	4	8	2	$(2,2,0)$
ω_5	**2**	4	**1**	**(1,1,0)**
ω_6	3	5	**1**	$(1,1,1)$
ω_7	3	7	2	$(2,1,0)$

Table 2 (Bold elements are the preferred interpretations)

Then we get: $[\Delta_\Sigma(E)] = \{\omega_1, \omega_5\}$, $[\Delta_{ws}(E)] = \{\omega_1\}$, $[\Delta_{max}(E)] = \{\omega_3, \omega_5, \omega_6\}$ and $[\Delta_{gmax}(E)] = \{\omega_5\}$.

As it can be seen in this example, max is the most cautious one, and is refined by $gmax$. Besides ws and Σ which correspond to other points of view, may select other interpretations.

2.3 Non-iteration (loss of associativity) of the process

The main drawback of Δ is the non-iteration of the process in a coherent way w.r.t. Dalal's distance even for associative operators. Indeed, in general $d(\omega, \Delta_f(K_1, K_2)) \neq f(d(\omega, K_1), d(\omega, K_2))$, where Δ_f is a merging operator based on an associative function f. This is due to the fact that when computing $d(\omega, \Delta_f(K_1, K_2))$, we lost the underlying priority between K_1 and K_2.

Let us now consider the above example and the majority operator which is associative. Let $K' = \Delta_\Sigma(\{K_1, K_2\})$. We can easily check that $d(\omega, \Delta_\Sigma(\{K_1, K_2\})) \neq d(\omega, K_1) + d(\omega, K_2)$. Therefore, it is not surprising to find $\Delta_\Sigma(\{K', K_3\}) \not\equiv \Delta_\Sigma(\{K_1, K_2, K_3\})$. Indeed, we have $\Delta_\Sigma(\{K', K_3\}) = \{\omega_1, \omega_3, \omega_5, \omega_7\}$.

3 Fusion of prioritized propositional bases

This section recalls the semantic and syntactic fusions of prioritized propositional bases developed in [2] in the framework of possibility theory. Let us first recall a minimal background on possibilistic logic (for more details see [4]). At the syntactic level, a possibilistic propositional base is a set of weighted formulas $B = \{(\phi_i, a_i) : i = 1, n\}$ where ϕ_i is a classical formula and a_i belongs to [0,1]. (ϕ_i, a_i) means that the certainty or necessity degree of ϕ_i is at least equal to a_i. We define the **a**-cut of B, denoted by $B_{\geq \mathbf{a}}$, the classical base $B_{\geq \mathbf{a}} = \{\phi_i : (\phi_i, a_i) \in B \; and \; a_i \geq \mathbf{a}\}$.

Given B, we can generate a unique possibility distribution, denoted by π_B such that all the interpretations satisfying all the propositions in B will have the highest possibility degree, namely 1, and the other interpretations will be ranked w.r.t. the highest necessity degree of propositions that they falsify, namely we get [4]: $\forall \omega \in \Omega$,

$$\pi_B(\omega) = \begin{cases} 1 & \text{if } \forall (\phi_i, a_i) \in B, \omega \in [\phi_i] \\ 1 - max\{a_i : \omega \notin [\phi_i]\} & \text{otherwise.} \end{cases}$$

This possibility distribution is such that $N(\phi_i) \geq a_i$ where N is a necessity measure defined from π_B [4].
We define by $Inc(B) = max\{a_i : B_{\geq a_i}\ is\ inconsistent\}$ the inconsistency degree of B. The useful consistent part of B is made of the formulas whose weights are above the inconsistency level. More formally, $\rho(B) = \{\phi_i : (\phi_i, a_i) \in B\ and\ a_i > Inc(B)\}$.
At the semantic level, the set of models of $\rho(B)$ corresponds to the interpretations having the highest possibility degree, namely $[\rho(B)] = \{\omega : \omega\ is\ maximal\ in\ \pi_B(\omega)\}$.
In [2], the authors suggest general syntactic approaches, which are semantically meaningful for fusing possibilistic bases. More precisely, let B_1, B_2 be two possibilistic bases, and π_1 and π_2 be their associated possibility distributions. Let $\oplus$ be a two place function whose domain is $[0,1] \times [0,1]$ to be used for aggregating $\pi_1(\omega)$ and $\pi_2(\omega)$. The only requirements for $\oplus$ are:

$i.$ $1 \oplus 1 = 1$,

$ii.$ If $a \geq c$, $b \geq d$ then $a \oplus b \geq c \oplus d$ (monotonicity).

The first one acknowledges the fact that if two sources agree that ω is fully possible, then the result of the combination should confirm it. The second property expresses that a degree resulting from a combination cannot decrease if the combined degrees increase.
In [2], it has been shown that the syntactic counterpart of fusing π_1 and π_2 is the following possibilistic base, denoted by $\mathcal{B}_\oplus$ and which is composed of:
- *the initial bases* with new weights defined by:
$\{(\phi_i, 1-(1-a_i)\oplus 1) : (\phi_i, a_i) \in B_1\} \cup \{(\psi_j, 1-1\oplus(1-b_j)) : (\psi_j, b_j) \in B_2\}$,
- and *the knowledge common* to B_1 and B_2 defined by:
$\{(\phi_i \vee \psi_j, 1-(1-a_i)\oplus(1-b_j)) : (\phi_i, a_i) \in B_1\ and\ (\psi_j, b_j) \in B_2\}$.

It has been shown that $\pi_{\mathcal{B}_\oplus}(\omega) = \pi_1(\omega) \oplus \pi_2(\omega)$ where $\pi_{\mathcal{B}_\oplus}$ is the possibility distribution associated to $\mathcal{B}_\oplus$.
Remarkable cases of $\oplus$ are the *minimum* (for short *min*), *maximum* (for short *max*) and *Product* (for short *Pro*). The first one is meaningful when the sources are consistent and may be not independent, the second one is appropriate when the sources are highly conflicting and the third one deals with independent sources. It has been shown, for these particular cases, that the fused possibilistic base is equivalent to:

$$\mathcal{B}_{min} = B_1 \cup B_2$$

$$\mathcal{B}_{max} = \{(\phi_i \vee \psi_j, min(a_i, b_j)) : (\phi_i, a_i) \in B_1, (\psi_j, b_j) \in B_2\}$$

$$\mathcal{B}_{Pro} = B_1 \cup B_2 \cup \{(\phi_i \vee \psi_j, a_i + b_j - a_i b_j) : (\phi_i, a_i) \in B_1\ and\ (\psi_j, b_j) \in B_2\}.$$

In the case of n sources, the syntactic computation of the resulting base can be easily applied when $\oplus$ is associative. The syntactic generalisation for non-associative operator ($\oplus$ is then a n-ary operator defined on vector of possibility distributions) is also possible.

4 Fusion of classical bases encoded in possibilistic logic

In Sections 2 and 3,we have presented merging operators for both classical and prioritized bases. In this section, we show that possibilistic logic can recover the boolean merging approaches reviewed in the previous sections. Let us first show how to associate a possibilistic base to a given classical base.

4.1 From a classical base to a possibilistic base

The aim of this subsection is given a classical propositional base K to construct a possibilistic base B_K such that:

$$\forall \omega, \pi_{B_K}(\omega) = f(d(\omega, K)).$$

First, we consider a base K which is a set of *literals* $K = \{a_i : i \in I\}$, namely K contains one formula composed of a conjunction of literals. To illustrate the construction of the possibilistic base associated to K, we use the base $K_3 = \{s \wedge d \wedge o\}$ of our example. Let us show how to construct B_{K_3}. Note that using Dalal distance, the worst interpretations are the ones which falsify all literals in K_3. At the syntactic level, the possibilistic base should contain the formula $(s \vee o \vee d)$ with the highest weight. Then, next preferred interpretations are those which only falsify one literal. This means that B_{K_3} contains the three formulas $(s \vee o)$, $(s \vee d)$, $(o \vee d)$ with a lower weight (which correspond to the three possible ways of removing one literal from the disjunction $s \vee o \vee d$). Next, the more preferred interpretations are those which falsify two literals. This leads to add in the possibilistic base the literals o, s, d with a smaller weight. To summarize, B_{K_3} should be of the form:
$B_{K_3} = \{(s \vee o \vee d, \alpha); (s \vee o, \beta); (s \vee d, \beta); (o \vee d, \beta); (s, \delta); (o, \delta); (d, \delta)\}$, with $\alpha > \beta > \delta$.
We can easily check that:
$\forall \omega, \pi_{B_{K_3}}(\omega) > \pi_{B_{K_3}}(\omega')$ iff $d(\omega, K_3) < d(\omega', K_3)$.
Moreover, if we let $\alpha = 1 - \varepsilon^3$, $\beta = 1 - \varepsilon^2$, $\delta = 1 - \varepsilon$, where ε is a very small number, then

- B_{K_3} is the result of combining the three elementary bases $\Sigma_s = \{(s, 1-\varepsilon)\}$, $\Sigma_o = \{(o, 1-\varepsilon)\}$ and $\Sigma_d = \{(d, 1-\varepsilon)\}$ with the product operator.
- $\forall \omega \in \Omega, \pi_{B_{K_3}}(\omega) = \varepsilon^{d(\omega, K_3)}$.

These remarks can be generalized. First, we need the following:

Lemma 1 *Let $K = \{a_i : i = 1, n\}$ be a set of literals. Let $\Sigma_i = \{(a_i, 1-\varepsilon)\}$ be n possibilistic bases with one formula. Then combining Σ_i's with the product operator leads to the following base*

$$B_K = \{(D_j(K), 1 - \varepsilon^j) : j = 1, n\},$$

where $D_j(K)$ is a disjunction of size j from K.

Example (continued)
Let us consider the base $K_3 = \{s, d, o\}$.
We have the combination of Σ_s and Σ_d with the product leads to:
$\{(s, 1-\varepsilon)\} \cup \{(d, 1-\varepsilon)\} \cup \{(s \vee d, 1-\varepsilon^2)\}$,
combining again this resulting base with Σ_o leads to:
$\{(s, 1-\varepsilon)\} \cup \{(d, 1-\varepsilon)\} \cup \{(o, 1-\varepsilon)\} \cup \{(s \vee d, 1-\varepsilon^2); (s \vee o, 1-\varepsilon^2); (d \vee o, 1-\varepsilon^2)\} \cup \{(s \vee d \vee o, 1-\varepsilon^3)\}$.

The following proposition explicits the encoding of Dalal's distance in possibilistic logic.

Proposition 1 *Let K be a set of literals. Let B_K be the base obtained using Lemma 1. Then,*

$$\pi_{B_K}(\omega) = \varepsilon^{d(\omega, K)}.$$

Now consider the case where K is a *general* knowledge base which is put under a disjunctive normal form i.e., $K = C_1 \vee ... \vee C_n$.
Note first that [9]: $d(\omega, K) = min_{i=1,n} d(\omega, C_i)$ (1)
Moreover, from Proposition 1, it is possible to associate to each C_i a possibilistic base B_{C_i} where
$\forall \omega, \pi_{B_{C_i}}(\omega) = \varepsilon^{d(\omega, C_i)}$ (2)
Then from (1) and (2) it can be easily guessed that:
$\pi_{B_K}(\omega) = max_{i=1,n} \pi_{B_{C_i}}(\omega) = max_{i=1,n} \varepsilon^{d(\omega, C_i)} = \varepsilon^{min_{i=1,n} d(\omega, C_i)} = \varepsilon^{d(\omega, K)}$.
Therefore the possibilistic base B_K associated to K is the result of combining the possibilistic bases B_{C_i}'s associated to each conjunct C_i of K with the maximum operator.

Proposition 2 *Let $K = \{C_1 \vee \cdots \vee C_n\}$ be a classical base, and B_{C_i}'s be the possibilistic bases associated to C_i's obtained from*

Lemma 1. Let B_K be the combination of B_{C_i}'s with the maximum operator. Then,

$$\pi_{B_K}(\omega) = \varepsilon^{d(\omega,K)}.$$

Now, due to the particular structure of B_{C_i}'s, the following proposition gives an equivalent rewriting of B_K, without computing explicitly B_{C_i}'s:

Proposition 3 *Let $K = \{C_1 \vee ... \vee C_n\}$ be a classical base. Let $m = min_{i=1,n}(|C_i|)$. Then, B_K is equivalent [2] to the following possibilistic base:*

$$\{(D_j, 1-\varepsilon^j) : j = 1, m\},$$

where D_j is a clause (i.e., disjunction of literals) containing exactly j literals from each C_i of K.

The number m in the previous proposition corresponds to the number of layers in the possibilistic base associated to K.

Example (continued) Let us put the bases K_1, K_2 and K_3 in a disjunctive form: $K_1 = \{C_1 \vee C_2\}$; $K_2 = \{C_3 \vee C_4\}$ and $K_3 = \{C_5\}$ such that $C_1 = \{s, \neg d\}$; $C_2 = \{o, \neg d\}$; $C_3 = \{\neg s, d, \neg o\}$; $C_4 = \{\neg s, \neg d, o\}$ and $C_5 = \{s, d, o\}$.
For K_1, we have $m = |C_1| = |C_2| = 2$. From Proposition 3:
$B_{K_1} = \{(s \vee \neg d \vee o, 1-\varepsilon^2); (s \vee \neg d, 1-\varepsilon); (s \vee o, 1-\varepsilon);$
$(\neg d \vee o, 1-\varepsilon); (\neg d, 1-\varepsilon)\}$ which is equivalent to $\{(s \vee \neg d \vee o, 1-\varepsilon^2); (s \vee o, 1-\varepsilon); (\neg d, 1-\varepsilon)\}$.
The formulas $(s \vee \neg d, 1-\varepsilon)$ and $(\neg d \vee o, 1-\varepsilon)$ are removed from the possibilistic base since they are subsumed [3] by $(\neg d, 1-\varepsilon)$.
With a similar computation we get:
$B_{K_2} = \{(\neg s \vee d \vee o, 1-\varepsilon^2); (\neg s \vee \neg d \vee \neg o, 1-\varepsilon^2);$
$(\neg s, 1-\varepsilon); (d \vee o, 1-\varepsilon); (\neg d \vee \neg o, 1-\varepsilon)\}$.
B_{K_3} has been previously computed.

4.2 Possibilistic encoding of the global distance

4.2.1 Semantic encoding of the global distance

In this subsection, given an information set $E = \{K_1, \cdots, K_n\}$, a boolean merging operator Δ, and $\mathcal{B} = \{B_{K_1}, \cdots, B_{K_n}\}$ the possibilistic counterpart of E obtained from Proposition 3, we show that it is possible to define a possibilistic merging operator $\oplus$ applied on $\mathcal{B}$ and yielding to $\mathcal{B}_\oplus$ such that $\pi_{\mathcal{B}_\oplus}$, the possibility distribution associated to $\mathcal{B}_\oplus$, is a function of d_Δ. Namely, $\pi_{\mathcal{B}_\oplus}(\omega) = f(d_\Delta(\omega, E))$.

Proposition 4 *Let $E = \{K_1, \cdots, K_n\}$ be an information set, and $\Delta_\Sigma(E)$ be the result of combining K_i's with the majority operator. Let $\mathcal{B} = \{B_{K_1}, \cdots, B_{K_n}\}$ be the possibilistic counterpart of E using Proposition 3, and $\pi_{B_{K_i}}$'s be the possibility distribution associated to B_{K_i}'s. Let $\pi_{\mathcal{B}_{Pro}}$ be the result of combining $\pi_{B_{K_i}}$'s with the product operator Pro. Then,*

$$\pi_{\mathcal{B}_{Pro}}(\omega) = \varepsilon^{d_\Sigma(\omega,E)}.$$

The proof can be easily checked. Recall that $d_\Sigma(\omega, E) = \Sigma_{i=1,n} d(\omega, K_i)$. Then, $\pi_{\mathcal{B}_{Pro}}(\omega) = \pi_{B_{K_1}} * \cdots * \pi_{B_{K_n}} = \varepsilon^{d(\omega,K_1)} * \cdots * \varepsilon^{d(\omega,K_n)} = \varepsilon^{\Sigma_{i=1,n} d(\omega,K_i)} = \varepsilon^{d_\Sigma(\omega,E)}$.
In a similar way we get the possibilistic counterparts to max and ws given in Table 3.
We now provide the encoding of $gmax$ operator. This is done by first giving a rewriting of the lexicographical order using the operation sum, after a transformation of a scale. More precisely, for each $d(\omega,K_i)$ we define $L_i = N^{d(\omega,K_i)}$ where N is a very large number [4]. Let $d_i = d(\omega, K_i)$ and $d'_i = d(\omega', K_i)$.
Then, we have the following lemma:

Lemma 2 *Let $d_\omega = (d_1, \cdots, d_n)$ and $d_{\omega'} = (d'_1, \cdots, d'_n)$ be two sequences of integers given in a decreasing order. Let $L = (L_1, \cdots, L_n)$ and $L' = (L'_1, \cdots, L'_n)$ be such that $L_i = N^{d_i}$ and $L'_i = N^{d'_i}$. Then,*

$$d_\omega <_{Lex} d_{\omega'} \Leftrightarrow \Sigma_{i=1}^n L_i < \Sigma_{i=1}^n L'_i.$$

Given this lemma, we can now give the encoding of $gmax$.
Let $K = \{C_1, \cdots, C_n\}$.
1- By Proposition 3, we have

$$B_K = \{(D_j, 1-\varepsilon^j) : j = 1, m\},$$

where D_j is a clause (i.e., a disjunction of literals) containing exactly j literals from each C_i of K.
2- We proceed to transforme the scale by defining from B_K a new base B'_K such that

$$B'_K = \{(D_j, 1-\varepsilon^{N_i}) : (D_j, 1-\varepsilon^j) \in B_K\}.$$

We can check that the possibility distribution associated to B'_K is as follows:

$$\pi_{B'_K}(\omega) = \begin{cases} 1 & \text{if } d(\omega, K) = 0 \\ \varepsilon^{N^{d(\omega,K)}} & \text{otherwise.} \end{cases}$$

Then, we have the following proposition:

Proposition 5 *Let $E = \{K_1, \cdots, K_n\}$ be an information set, and $\mathcal{B} = \{B_{K_1}, \cdots, B_{K_n}\}$ be the syntactic counterpart to E from Proposition 3. Let B'_{K_i}'s be the bases from B_{K_i}'s using Step 2 above. Let $\pi_{B'_{K_i}}$ be the possibility distribution associated to B'_{K_i}. Let $\pi_{\mathcal{B}_{Pro}}$ be the result of combining $\pi_{B'_{K_i}}$'s with Pro operator. Then,*

$$\omega >_{gmax} \omega' \Leftrightarrow \pi_{\mathcal{B}_{Pro}}(\omega) > \pi_{\mathcal{B}_{Pro}}(\omega').$$

The following table summarizes the above propositions. It gives which possibilistic operator $\oplus$ should be used for recovering boolean merging operator Δ:

Δ	$\oplus$
max	min
Σ	Pro
ws	weighted Pro
$gmax$	Pro (with a suitable scale)

Table3

where weighted Pro is simply the product applied repeatedly. Since the α_i's are integers, each base is repeated n times if $\alpha_i = n$.

4.2.2 Syntactic counterpart of the semantic encoding

In this subsection we show that once the possibilistic bases associated to K_i's are computed, the syntactic computation of $\Delta(E)$ is immediate using the syntactic merging operators applied to possibilistic bases.
At the semantic level, we have already shown that given an information set $E = \{K_1, \cdots, K_n\}$ and a boolean operator Δ, we can define $\mathcal{B} = \{B_{K_1}, \cdots, B_{K_n}\}$ such that combining $\pi_{B_{K_i}}$'s (the possibility distributions associated to B_{K_i}'s) with some $\oplus$, the counterpart of Δ according to Table 3, leads to the following relation (when $\Delta \in$

[2] The equivalence is understood in the sense that the two possibilistic bases generate the same possibility distributions.
[3] $(\phi, a) \in B$ is said to be subsumed by B if it can be entailed from all the formulas (different from (ϕ, a)) having a weight at least equal to a. It can be checked that B and $B - \{(\phi, a)\}$ are equivalent (see [4]).
[4] N should be s.t. for each k ($k > 0$), we have $N^k > \Sigma_{k'<k} N^{k'}$.

$\{\Sigma, max, ws\}$): $\pi_{\mathcal{B}_\oplus}(\omega) = \varepsilon^{d_\Delta(\omega,E)}$ (3)
Let $\Delta(E)$ be the result of combining K_i's with Δ.
We know that the models of $\Delta(E)$ are the interpretations which minimize $d_\Delta(\omega, E)$. Then, from (3), these interpretations maximize $\pi_{\mathcal{B}_\oplus}$ since $\varepsilon < 1$.
For $gmax$, we have shown that $\omega >_{gmax} \omega' \Leftrightarrow \pi_{\mathcal{B}_{Pro}}(\omega) > \pi_{\mathcal{B}_{Pro}}(\omega')$. Here, we also have the preferred interpretations w.r.t. $>_{gmax}$ are those which maximize $\pi_{\mathcal{B}_{Pro}}$.
Given a possibility distribution π_B associated to a possibilistic base B, we also know that the interpretations which maximize π_B are the models of $\rho(B)$, the consistent part of B (see Section 3).
Now, $\pi_{\mathcal{B}_\oplus}$ is the result of combining B_{K_i}'s with $\oplus$. Hence, the preferred interpretations w.r.t. $\pi_{\mathcal{B}_\oplus}$ are the models of $\rho(\mathcal{B}_\oplus)$.

Proposition 6 *Let $E = \{K_1, \cdots, K_n\}$ be an information set, and $\Delta(E)$ be the result of combining K_i's with Δ. Let $\mathcal{B} = \{B_{K_1}, \cdots, B_{K_n}\}$ be the possibilistic counterpart of E using Proposition 3. Let $\oplus$ be the syntactic counterpart of Δ obtained from Table 3, and $\mathcal{B}_\oplus$ be the result of combining B_{K_i}'s with $\oplus$. Then,*

$$[\Delta(E)] = [\rho(\mathcal{B}_\oplus)].$$

Example (continued) Let us consider the majority operator Σ. Let B_{K_1},B_{K_2} and B_{K_3} be the possibilistic counterpart of K_1, K_2 and K_3 computed in the last example. Then to compute the possibilistic base associated to the combination of $B_{K_1}, B_{K_2}, B_{K_3}$ with the majority operator, we combine B_{K_i}'s with the product operator. Since the product operator is associative, let us first compute $\mathcal{B}'$ the base resulting from the combining B_{K_1} and B_{K_2}.
$\mathcal{B}' = \{(\neg s \vee \neg d \vee \neg o, 1-\varepsilon^3); (s \vee d \vee o, 1-\varepsilon^2); (\neg s \vee \neg d, 1-\varepsilon^2); (\neg d \vee \neg o, 1-\varepsilon^2); (s \vee \neg d \vee o, 1-\varepsilon^2); (\neg s \vee d \vee o, 1-\varepsilon^2); (s \vee o, 1-\varepsilon); (\neg d, 1-\varepsilon); (\neg s, 1-\varepsilon); (d \vee o, 1-\varepsilon)\}$.
Then combining $\mathcal{B}'$ and B_{K_3}, after removing subsumed formulas, leads to: $\mathcal{B}_\oplus = \{(s \vee d \vee o, 1-\varepsilon^5); (s \vee \neg d \vee o, 1-\varepsilon^4); (\neg s \vee d \vee o, 1-\varepsilon^4); (\neg s \vee \neg d \vee o, 1-\varepsilon^3); (s \vee \neg d \vee \neg o, 1-\varepsilon^3); (\neg s \vee \neg d \vee \neg o, 1-\varepsilon^3); (\bot, 1-\varepsilon^2)\}$.
Note that we obtain a contradiction here with a weight $1-\varepsilon^2$ which expresses conflict between sources.
Note that $Inc(\mathcal{B}_\oplus)$ is always of the form $1-\varepsilon^j$ where j is the minimal weight in the global distance obtained from merging E by Δ. For instance, we can check that in this example we have $Inc(\mathcal{B}_\oplus) = 1-\varepsilon^2$ and indeed 2 is the minimal distance in Table 2 (for Σ). When $j = 0$ namely $Inc(\mathcal{B}_\oplus) = 0$, this simply means that $\wedge E = K_1 \wedge ... \wedge K_n$ is consistent.
Now, to recover $\Delta_\Sigma(\{K_1, K_2, K_3\})$ we simply compute $\rho(\mathcal{B}_\oplus)$, namely $\rho(\mathcal{B}_\oplus) = \{s \vee d \vee o, s \vee \neg d \vee o, \neg s \vee d \vee o, \neg s \vee \neg d \vee o, s \vee \neg d \vee \neg o, \neg s \vee \neg d \vee \neg o\} \equiv \{\neg d, o\}$.
Clearly we can check that $[\rho(\mathcal{B}_\oplus)] = \{\omega_1, \omega_5\}$ which has been obtained in the examples above.

Remark 1 *Note that the advantage of giving a possibilistic base instead of the classical base $\rho(\mathcal{B}_\oplus)$ as a result of the merging is the capability of iteration for associative operators, which is not possible with classical merging operators. Indeed, in the above example we have $(B_{K_1} \oplus B_{K_2}) \oplus B_{K_3} = B_{K_1} \oplus B_{K_2} \oplus B_{K_3}$, and this is general.*

4.3 Integrity constraints

We consider now the merging of classical knowledge bases with integrity constraints. The integrity constraints are requirements that the merged base must satisfy. Let $E = \{K_1, ..., K_n\}$ be the information set to merge, and μ be a set of constraints. We denote by $\Delta(\mu, E)$ the result of merging with respect to an operator Δ. The operation of fusion follows the same steps as Δ. The only change appears in selecting models of $\Delta(\mu, E)$ which focuses on models of μ which are minimal in $\leq_E^\Delta$, namely

$$[\Delta(E)] = min([\mu], \leq_E^\Delta)$$

It means that we only consider interpretations which satisfy the set of integrity constraints. Since the formulas of μ must be satisfied, they are considered as fully reliable in possibilistic framework. The possibilistic propositional base associated to a set of integrity constraints is then: $B_\mu = \{(\phi_i, 1) : \phi_i \in \mu\}$.

Proposition 7 *Let E and μ be respectively an information set and a set of integrity constraints, and Δ be a boolean merging operator. Let $\mathcal{B}_\oplus$ be the possibilistic counterpart of E. Then the possibilistic base associated to E and μ is simply $\mathcal{B}_\oplus \cup B_\mu$.*

5 Conclusion

The paper has shown how boolean merging operators can be modelled in the possibilistic logic framework. This has several benefits. First, as already said we can iterate the merging operator in a coherent way w.r.t. priorities. Second, classical merging approaches heavily lie on the use of Dalal's distance. However, as in our example where a student says that he would like to learn SQL or O_2, it is not necessarily the case that he wants to give the same priority to both. He may prefer SQL to O_2; such a disymmetry between the literals can be easily represented by entering from the beginning priorities in the possibilistic logic framework. Third, with the exception of [9] which has provided a syntactic computation in the case of a weighted sum, classical methods are only semantically defined. Possibilistic logic offers a general syntactic computation machinery for the merging process. Such computations can be still simplified if one only looks for producing classical bases without priorities, since in this case we only have to generate the consistent part of the possibilistic base. A postulate-based study of fusion in the possibilistic framework can be found in [1].

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