Kalman-like filtering and updating in a possibilistic setting

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Abstract: This paper proposes a qualitative counterpart of Kalman filtering in the possibilistic logic setting. It corresponds to a type of updating involving a prediction step followed by a revision step. This is compared with updating operations based on imaging in the sense of Lewis. Imaging is reconsidered in the perspective of a generalized view of Kalman filtering where it appears as a particular case of Kalman filtering. A syntactic counterpart of qualitative filtering is given in terms of weighted knowledge bases.

1. Introduction

Since the pioneering work of Alchourron, Gärdenfors and Makinson [1], and the publication of the seminal book "Knowledge in Flux" by Gärdenfors [21], there has been an important and increasing interest in the modelling of belief change in A. I. Progressively, basic distinctions have emerged between various types of belief change: revision of beliefs by an input information in a static world vs. update of beliefs in a dynamic world [22], revision by an input held as certain and prioritary vs. revision by an uncertain information [10, 4, 17], revision vs. focusing on a class of reference [18], revision of beliefs vs. revision of preferences [26, 3]. What is noticeable is that the same distinctions can be made in various representational settings provided that these frameworks, which might be symbolic or numerical [24, 20], are rich enough for enabling the expression of these distinctions.
Another important aspect with respect to belief revision is the epistemic entrenchment underlying any well-behaved revision process, which should obey Alchourron, Gärdenfors and Makinson (AGM) postulates. Since an epistemic entrenchment relation is closely related to a necessity measure in the sense of possibility theory [15], the framework of possibilistic logic [13] enables us to envisage belief revision both at the syntactic level of a possibilistic logic base, and in an equivalent manner, at the semantic level of a possibility distribution ranking the interpretations. In this approach the ordering on which the revision is based is explicitly associated with the formulas and is modified in the revision process. This view is also advocated by Williams [29] in her related approach based on adjustments.
The present paper should be understood in this general perspective, where different types of belief change operations have been investigated both at the semantic and at the syntactic level. A qualitative counterpart of a well-known "updating" method, Kalman filtering (briefly recalled in Section 2), is introduced in Section 3 and compared to updating based on imaging in Sections 4 and 5, in the setting of possibility theory. Then a syntactic counterpart of these machineries is outlined in Section 6.
A preliminary draft of this paper was made electronically available at http://www.lucs.lu.se/spinning/ in September 1999 as a contribution to the Festschrift in honour of Peter Gärdenfors on the occasion of his fiftieth birthday. The present paper extends and improves this draft in various respects.

2. Kalman filtering and dynamic estimation

Kalman filtering is the basis of well-known updating techniques in systems engineering (e.g., [2]), in the case of an evolving system when events are dated. The idea underlying Kalman filtering, namely a two-steps procedure involving prediction followed by revision, can be of interest in other settings. Recently, Castel, Cossart and Tessier [5], Cossart and Tessier [8] have proposed to transpose these ideas in a symbolic setting for a situation assessment problem. Let us first consider the probabilistic framework. We only give here an abstract view of Kalman filtering. Let Ω be a set of interpretations. ω Ω is also called a state or a possible world, or also an elementary event. Subsets of Ω are called propositions or formulas, or simply events. It is assumed that there exists a prediction function f such that $f(\omega_t) = \omega_{t+1}$, where ω_t Ω is the state at time t and $f(\omega_t)$ is the resulting state at time t+1. Knowing the probability distribution p_t on the system state at time t, the prediction (forecast distribution) at t + 1 is given in ω by:

$$p'(\omega) = P_t(f^{-1}(\omega)). \qquad (1)$$

where $f^{-1}(\omega) = \{\omega' : \omega = f(\omega')\}$. Let $A \prod \Omega$ be an information available at time t+1, the updated state at t+1 using Bayes rule, is

$$p_{t+1}(\omega) = p'(\omega | A). \qquad (2)$$

These two equations (1) and (2) can be equivalently written

$$p_{t+1}(\omega) = P_t(f^{-1}(\omega) \mid A) = P_t(f^{-1}(\omega)) / P_t(f^{-1}(A)) \text{ if } \omega \quad A.$$
$$= 0 \qquad \text{otherwise} \qquad (3)$$

Thus this type of updating is decomposed into a prediction step followed by a revision or a conditioning step. The underlying idea is that the prediction of the next state at t+1 pervaded with uncertainty is improved by taking into account the observation A. Classical Kalman filtering is a particular case of this view where Ω is a continuous state space, f is a linear function, and Gaussian distributions are used.

3. Possibilistic filtering

A brief background on possibility theory is first given in a belief change perspective, before proposing a possibilistic counterpart of Kalman filtering.

3.1. The possibility theory setting

Possibility theory [31] provides a framework for uncertainty modelling, which can be numerical or remain qualitative, and which departs from probability by the use of maxitive (rather than additive) law and the existence of a dual pair of measures for

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assessing the uncertainty. See [19] for a detailed overview of possibility theory. The possibilistic approach enriches the knowledge representation provided by the pure logical setting from the point of view of expressiveness. Instead of viewing a belief state as a flat set Ω of mutually exclusive states, one adds a complete partial ordering on top of the logical structure, according to which some states are considered as more plausible than others. A cognitive state can then be modelled by a possibility distribution π, that is, a mapping from Ω to a totally ordered set V containing a greatest element (denoted 1) and a least element (denoted 0), typically the unit interval V = [0,1]. However any finite, or infinite and bounded, chain will do as well. This approach is also close to Spohn [28]'s well-ordered partitions, see [15].

A consistent cognitive state π is such that $\pi(\omega) = 1$ for some ω, i.e., at least one of the states is considered as completely possible in Ω. In such a case π is said to be normalized. Here consistency can be a matter of degree. A cognitive state π is said to be partially inconsistent if $0 < \max_{\omega \quad \Omega} \pi(\omega) < 1$. When $\pi(\omega)>\pi(\omega')$ then ω is a more plausible state than ω'. A possibility measure $\prod$ is associated with a possibility distribution π, namely:

$$\prod(A) = \sup_{\omega \quad A} \pi(\omega).$$

Possibility measures thus satisfy the following characteristic decomposition property: $\prod(A \approx B) = \max(\prod(A), \prod(B))$.

Necessity measures N are defined by duality, namely

$$N(A) = 1 - \prod(\neg A) \text{ and } N(A \leftrightarrow B) = \min(N(A), N(B)).$$

Let us give the definition of conditioning which transforms a cognitive state π into a possibility distribution $\pi^*_A = \pi(\cdot \mid A)$ obtained by revising π with input A:

$$\pi(\omega \mid A) \quad = 1 \quad \text{if } \pi(\omega) = \prod(A),\ \omega \quad A$$
$$= \pi(\omega) \quad \text{if } \pi(\omega) < \prod(A),\ \omega \quad A \qquad (4)$$
$$= 0 \quad \text{if } \omega \quad A.$$

3.2. Filtering in the possibilistic framework

Let us now give the possibilistic counterpart of Kalman filtering first suggested in [14]. Let f be a prediction function $f(\omega_t) = \omega_{t+1}$, where ω_t is the state at time t. Knowing the possibility distribution π_t on the system state at t and the input information A available at time t+1, the updated state at t+1 can be computed in two steps using possibilistic conditioning:

$$\pi'(\omega) = \prod_t(f^{-1}(\omega)) = \max_{\omega' \quad f^{-1}(\omega)} \pi_t(\omega') \qquad (5)$$

and $\qquad \pi_{t+1}(\omega) = \pi'(\omega \mid A). \qquad (6)$

Note that π_{t+1} is always normalized (if π_t is). In the above formula, it would be possible to replace $\pi'(. \mid A)$ by a more general expression in case of an uncertain observation (A, α). See [17] for conditioning by an uncertain input.

More generally, one may consider a family $\{\pi_\omega, \omega \quad \Omega\}$ describing a transition graph, hence generalizing f as a fuzzy relation R, such that $\mu_R(\omega,\omega') = \pi_\omega(\omega')$ is the plausibility that ω' follows ω, and then compute the image of the cognitive state pertaining to the initial state through the fuzzy relation R (prediction) and revise the so-obtained prediction by the input, that is compute the updated possibility distribution π_{t+1}

$$\pi'(\omega) = \max_{\omega'} \min(\pi_t(\omega'), \pi_{\omega'}(\omega)) . \qquad (7)$$

and $\qquad \pi_{t+1}(\omega) = \pi'(\omega \mid A). \qquad (8)$

Note that π_{t+1} is normalized provided that $\exists \omega, \omega'$ such that $\pi_t(\omega')=1$ and $\pi_{\omega'}(\omega)=1$. Clearly, (7) generalizes (5) by letting $\pi_{\omega'}(\omega) = 1$ if $f(\omega') = \omega$ and 0 otherwise.

4 - Updating

In the following, updating precisely refers to the belief change operation which aims at restoring uptodate views of the world in a dynamic world when receiving new information. At the theoretical level probabilistic imaging belongs to this type of operation. We then consider its possibilistic counterpart.

4.1 Probabilistic imaging

Another path in the problem of probabilistic change, which departs both from conditioning and filtering, is the one followed by Lewis [25]. Assume that the set Ω of possible states possesses a distance measure and is such that for any state $\omega \quad \Omega$, and any set $A \prod \Omega$, there is a single state ω_A in A defined as the closest state to ω. If there is no natural distance on Ω, we may think of using Dalal's [9] distance. Then the principle of minimal change upon learning that some event $A \prod \Omega$ has occurred can be expressed as an advice to allocate the probability weight of each state that becomes impossible to the closest state that is made possible by the input. The input is here at the same level of generality as the prior probability, and the translation of worlds expresses that the current state has changed, and not that our previous beliefs about it were wrong. This updating rule can be formally expressed as

$$\forall \omega \quad A,\ p_A(\omega) = \sum_{\omega':\ \omega=\omega'_A} p(\omega'). \qquad (9)$$

This rule is called 'imaging' because p_A is the image of p on A obtained by moving the masses $p(\omega')$ for ω' A to ω'_A A, with the natural convention that $\omega'_A = \omega'$ if ω' A. This rule actually comes from the study of conditional logics[23], and was motivated by the study of the probability of a conditional in such logics.

The imaging rule has been generalized by Gärdenfors [21] to the case when the set of states in A closest to a given state ω contains more than one element. If $A(\omega) \prod A$ is the subset of closest states from ω, $p(\omega)$ can be shared among the various states ω' $A(\omega)$ instead of being allocated to a unique state. Clearly, instead of sharing $p(\omega)$ among ω' $A(\omega)$, a less committed update is to allocate $p(\omega)$ to $A(\omega)$ itself (and none of its subsets). In that case the imaging process produces a basic probability assignment [27] in the sense of Dempster [11]'s view of belief functions. But this type of update is not consistent with Bayesian probabilities because the result of imaging is a family of probability distributions, and not a unique one.

Note that imaging can turn impossible states into possible ones, i.e., one may have $p_A(\omega) > 0$ while $p(\omega) = 0$ for some ω, e.g., if ω_A is such that $p(\omega_A) = 0$. As a consequence a sure fact B a priori, i.e., such that P(B) = 1 may become uncertain, i.e., $P_A(B) < 1$. This is not the case with Bayesian conditioning. In order to preserve this kind of monotonicity property, one idea (see [21]) is to build P_A as the image of P on $A \leftrightarrow S$ where $S = \{\omega \mid P(\omega) > 0\}$ is the support of P. However, as with the Bayesian rule, $P(A) = 1$ □ $P_A = P$; this is the probabilistic version of the success postulate of Katsuno and Mendelzon [22] for updating. In fact, all postulates of Katsuno and Mendelzon hold or have a natural counterpart for probabilistic cognitive states, except the postulate which expresses that the conjunction of B with the result of an updating by A entails the result of the updating by the conjunction of A and B (see, e.g., [24]).

4.2 Possibilistic imaging

It is easy to define the possibilistic counterpart to Lewis' imaging since this type of belief change is based on mapping each possible state to the closest one that accommodates the input information. As above, define for any $\omega \in \Omega$, and non-empty set $A \subseteq \Omega$ the closest state to ω where A is true, that is, where $\omega_A \in A$. Then the image $\pi°_A$ of a cognitive state π in A is

$$\pi°_A(\omega) = \max_{\omega': \omega=\omega'_A} \pi(\omega') \text{ if } \omega \in A$$
$$= 0 \text{ if } \omega \notin A. \quad (10)$$

If there is more than one state ω'_A closest to ω', then the weight $\pi(\omega')$ is allocated to each of the closest states forming the set $A(\omega')$, and the above imaging rule becomes

$$\pi°_A(\omega) = \max_{\omega': \omega \in A(\omega')} \pi(\omega') \quad \text{if } \omega \in A$$
$$= 0 \text{ if } \omega \notin A. \quad (11)$$

Note that $\pi°_A$ is normalized if π is normalized. Defining $\forall\omega$, $A(\omega)$ precisely as $\{\omega' \mid \pi(\omega') = \Pi(A)\}$, which does not depend on ω, then $\pi°_A = \pi(\cdot \mid A)$, i.e., we recover the revision based on conditioning. Clearly in this setting, we see that possibilistic imaging formally subsumes the AGM revision. However this link is somewhat artificial. Indeed imaging can be envisaged in a dynamic perspective in which $A(\omega)$ represents the states where A is true that most plausibly follow ω. Clearly $A(\omega)$ depends on the current system state ω. Then input A warns the agent that a change in that system state has occurred.

It is easy to check that the above updating rule defined by (10) satisfies all postulates of Katsuno and Mendelzon [22]'s updates (see [14]). Katsuno and Mendelzon [22] have proved that any change operation that obeys all postulates involves a proximity structure on Ω, that is, a family $\{<_\omega, \omega \in \Omega\}$ of partial ordering relations, where $\omega'' <_\omega \omega'$ means that ω'' is closer than ω' to ω. In a dynamic system perspective, a state is the state of a dynamic system and $\{<_\omega, \omega \in \Omega\}$ represents a partial transition graph where $\omega'' <_\omega \omega'$ means that ω'' is a more plausible successor to ω than ω'. Then $A(\omega)$ gathers all states in A that are minimal in the sense of $<_\omega$.

It has been shown in [12] that adding one more postulate the proximity structure on Ω is a family $\{\leq_\omega, \omega \in \Omega\}$ of *complete* preordering relations, that can be equivalently represented by a family $\{\pi_\omega, \omega \in \Omega\}$ of qualitative possibility distributions. Then the most plausible states in A reachable from ω form the set $A(\omega) = \{\omega' \in A, \pi_\omega(\omega') = \Pi_\omega(A)\}$ where Π_ω is the possibility measure associated to π_ω. Defining R_A as the relation that to each ω assigns its closest neighbours $A(\omega)$ in A, the above update formula (10) is nothing but Zadeh [30]'s extension principle that characterizes the fuzzy image of the fuzzy set whose membership function is π. Namely, if $\pi = \mu_F$ then $\pi°_A = \mu_{R_A \circ F}$ with $\mu_{R_A}(\omega,\omega') = 1$ if $\omega' \in A(\omega)$ and $\mu_{R_A}(\omega,\omega') = 0$ otherwise and $\mu_{R_A \circ F}(\omega') = \max_\omega \min(\pi(\omega), \mu_{R_A}(\omega,\omega'))$. In other words, the uncertainty on the initial system state is propagated over to the next state via the input-dependent prediction relation based on the transition graph.

More generally, let $\{\pi_\omega, \omega \in \Omega\}$ be a family describing a transition graph. The only requirement on distributions of this family is that they should satisfy the generalized inertia principle:

$$\pi_{\omega'}(\omega)=1 \Leftrightarrow \omega = \omega' \quad (12)$$

We can compute the image of the cognitive state pertaining to the initial state through the fuzzy relation $\{\pi_\omega, \omega \in \Omega\}$. The updated possibility distribution $\pi°_A$ is computed in two steps:

$$\hat{e}\pi(\omega) = \max_{\omega'} \min(\pi(\omega'), \pi_{\omega'}(\omega)) \quad (13)$$
$$\pi°_A(\omega) = \hat{e}\pi(\omega \mid A). \quad (14)$$

This can be viewed as a generalized form of update. Note that if ω and ω' are such that $\pi(\omega') = 1$ and $\pi_{\omega'}(\omega)=\Pi_{\omega'}(A)$ then $\pi°_A(\omega)=1$ while in Kalman filtering $\pi_{t+1}(\omega)<1$ if there exists $\omega'' \in A$ such that $\hat{e}\pi_{t+1}(\omega'')>\pi_{t+1}(\omega)$. This situation occurs if the transition to ω'' (from a highly plausible state different from ω') is more plausible than the transition from ω' to ω. This type of update operation can be encountered in other settings [6, 7].

5. Filtering vs. Imaging

We first highlight the differences between imaging and Kalman filtering, and then we show how imaging can be encoded as a Kalman-like filtering.

Clearly, filtering and imaging use equations presenting strong similarities in order to compute the new cognitive state after learning some new event A. The basic difference is that in Kalman filtering, any prediction function can be used, and it does not depend on the event A. However, in imaging the distance is a strong constraint since if $\omega \in A$ then the closest interpretation of ω in A is ω itself. This is clearly expressed by equation (12) when generalizing updating, while such requirement does not appear when generalizing Kalman filtering.

Moreover, in imaging no possible initial state in A ($\pi(\omega)>0$ and $\omega \in A$) is deemed impossible after A has occurred, since the used distance depends on A and is such that $\omega=\omega_A$ for $\omega \in A$. Imaging thus comes down in the probabilistic setting to computing $p_A(\omega) = P_t(f_A^{-1}(\omega))$ for all $\omega \in A$, and does not require any normalization since $P_A(A) = 1$.

In the possibilistic setting (as well as in probabilistic setting) we always have:

$$\pi°_A(\omega) \geq \pi(\omega) \quad \text{for } \omega \in A. \quad (15)$$

While $A(\omega)$ is a subset of A in the imaging, the value of the prediction function, and more generally π_ω, does not depend on A when filtering. Instead of selecting $A(\omega)$, generalized filtering considers the family $\{\pi_\omega, \omega \in \Omega\}$ describing the transition graph, as a fuzzy relation R such that $\mu_R(\omega,\omega')=\pi_\omega(\omega')$.

As a consequence of using a prediction function which does not depend on A, the above inequality (15) does not hold, and even worse one may have:

$$\pi_t(\omega) \geq 0 \text{ and } \pi_{t+1}(\omega)=0 \quad \text{for } \omega \in A. \quad (15)$$

This should not be viewed as a drawback since the prediction function is not a "similarity" measure.

It is clear that π_{t+1} in (6) differs from $\pi°_A$ in (11) because they correspond to different strategies. Using $\pi°_A$ the assumed transition from each state ω is always supposed to be the most plausible one(s) modelled by $A(\omega)$, and the intrinsic plausibility of this transition is not considered. Using π_{t+1}, transitions that are not the most plausible ones compatible with A are considered via π_{ω} and lead to possible final states that are neglected by imaging. Hence the two approaches are different. However it is obvious that imaging makes sense for answering questions about the next most plausible state, while the prediction/revision approach is more adapted to the handling of trajectories in the transition graph, and is the counterpart in the possibilistic setting of Kalman filtering.

From this discussion, it is easy to see that Kalman filtering is more general than updating since there is no restriction on the function f in Kalman-like filtering. Hence, the distance measure used in imaging can be encoded using some particular kind of transition functions. Indeed, let A be a subset of Ω and let d be some distance which gives for each interpretation ω its closest interpretation ω' in A. Then for each A, and for each d, we define $f_{A,d}$ in the following way:

$\forall\omega$, $f_{A,d}(\omega) = \omega'$ where ω' is the closest interpretation to ω in A w.r.t. d (16)

The converse does not hold. This is mainly due to the strong assumption imposed by the distance where if ω A then the closest interpretation of ω in A is ω itself.

6. Syntactic filtering

Filtering (and also updating) has been defined at the semantic level. In this section we provide its syntactic counterpart. We first give a compact representation of a possibility distribution by means of possibilistic knowledge bases.

6.1. Background on possibilistic logic

A possibilistic knowledge base is made up of a finite set of weighted formulas

$$\Re=\{(\phi_i, a_i), i=1,n\}$$

where a_i is understood as a lower bound on the degree of necessity $N(\phi_i)$. Formulas with zero degree are not explicitly represented in the knowledge base (only beliefs which are somewhat accepted by the agent are explicitly represented). The higher the weight, the more certain the formula. The weights a_i hence induce constraints on possibility distributions. Indeed, each pair (ϕ_i, a_i) imposes that the induced possibility distribution π should satisfy: $N(\phi_i)\geq a_i$. Let $\Sigma_{\varepsilon a i}$ be the set of formulas with weight at least equal to a_i. A possibilistic knowledge base $\Re$ is said be consistent if its classical counterpart, obtained by forgetting the weights, is classically consistent. We denote by

$$Inc(\Sigma) = \max\{a_i : \Sigma_{\varepsilon a i} \text{ is inconsistent}\}$$

the inconsistency degree of Σ. $Inc(\Re) = 0$ means that $\Sigma_{\varepsilon a i}$ is consistent for all a_i.

Given a possibilistic knowledge base $\Re$, we can generate a unique possibility distribution by associating to each interpretation, the level of compatibility with agent's beliefs, i.e., with $\Re$. This possibility distribution is such that all the interpretations satisfying all the beliefs in $\Re$ have the highest possibility degree, namely 1, and the other interpretations will be ranked w.r.t. the highest belief that they falsify [13]. The possibility distribution associated with a knowledge base $\Re$ is:

$\forall\omega$ Ω, $\pi_{\Re}(\omega) = 1$ if $\forall(\phi_i\ a_i)$ $\Re$, $\omega \bullet \phi_i]$

$= 1 - \max\{ a_i : (\phi_i\ a_i)$ $\Re$ and ω ° ϕ_i } otherwise.

The possibility distribution $\pi_{\Re}$ is not necessarily normalized, however $\pi_{\Re}$ is normalized iff Σ is consistent.

Lastly, syntactic possibilistic inference is very efficient with a complexity close to the one of classical logic.

6.2. Syntactic counterpart of conditioning

Let $\sum$ be a possibilistic knowledge base, and $\pi_{\Re}$ its associated possibility distribution (using the above definition). This subsection provides a syntactic counterpart of conditioning $\pi_{\Re}$ with some observation A. This consists in constructing from a possibilistic base Σ and the new information A, a new possibilistic base Σ' such that:

$$\forall\omega, \pi_{\Re'}(\omega) = \pi_{\Re}(\omega|A).$$

This is done in a very simple way: add the input A to the knowledge base with highest possible priority (i.e., 1); compute the level of inconsistency $x = Inc(\Sigma \approx \{(A, 1)\})$ of the resulting possibly inconsistent knowledge base; drop all formulas with priority less than or equal to this level of inconsistency. This guarantees that the remaining beliefs are consistent with A. More formally, $\sum'$ is defined as follows:

$$\sum' = \{(\phi_i, a_i) : (\phi_i, a_i)\ \ \sum \text{ and } a_i > x\} \approx \{(A, 1)\}.$$

6.3. Syntactic counterpart of filtering

Let $\sum_t$ be a knowledge base associated with π_t (using the above definition). We recall that given a prediction function f and a new observation A, the new possibility distribution is computed in two steps:

i) compute π' using the function f in the following way:

$$\pi'(\omega) = \max_{\omega':\omega=f(\omega')} \pi_t(\omega').$$

ii) apply conditioning of π' to A, namely: $\pi_{t+1}(\omega) = \pi'(\omega|A)$

Now we are interested in constructing $\sum_{t+1}$ such that :

$$\pi_{\sum_{t+1}}(\omega) = \pi_{t+1}(\omega).$$

Let $\sum_t$ be the possibilistic base associated with π_t. Let us construct $\sum'$ the possibilistic associated with π' obtained in step (i). Let $\alpha_n = 1 > \alpha_{n-1} > \ldots > \alpha_1$ (with let $\alpha_0=0$) as the weights used in $\sum_t$ and we denote by S_i be the set of classical formulas having the weight equal to α_i. We now describe π_t in terms of classes corresponding to the same certainty level, that we denote C_i, and defined as follows:

$C_0 = [S_1 \approx \ldots \approx S_n]$

$C_i = [S_{i+1} \approx \ldots \approx S_n] - [S_i \approx \ldots \approx S_n]$, for i=1,n-1,

$C_n = \{\text{countermodels of } S_n\}$,

where $[\phi]$ denotes classical models of ϕ. We can easily check that the C_i's encodes exactly the possibility distribution associated to $\sum_t$, namely we have: $\pi_t(\omega) = 1 - \alpha_i$ iff ω C_i .

We are taking advantage of the compatibility of the extension principle with the level cutting of $\sum_t$. Let us describe similarly π_{t+1} using classes E_i's such that ω E_i iff $\pi'(\omega) = 1 - \alpha_i$. Then we can easily check that E_i's can be defined using the classes C_i's, and the function f as follows:

$E_0 = f(C_0)$, $E_i = f(C_i) - \approx_{j=0,i-1} f(C_j)$, for i = 1, n-1 and

$E_n = \Omega - \approx_{j=0,n-1} f(C_j)$, where $f(C_i) = \{f(\omega) : \omega \; C_i\}$. Note that it may happen that some E_i's can be empty.

Given this representation, the knowledge base $\sum'$ associated with π' can be defined as follows : Let ξ_i be a classical formula whose counter-models is the set E_{n-i+1}. Then:

$$\sum' = \{(\xi_i, \alpha_i) : i=1,n\}.$$

Note that a more efficient construction of $\sum'$ can be obtained if the function f is directly defined on formulas rather than on interpretations. In some cases, f may directly available on the set of varaibles. Let us illustrate this idea on a simple example. Consider a moving object in a discritized space whose positions denoted by a_1, a_2, a_3, a_4 form a partition. The initial position of the object O is described by the possibilistic knowledge base (we omit mutual exclusiveness constraints on the ai's):

$$\sum = \{(a_1 \Delta a_2 \Delta a_3, 1), (a_1 \Delta a_2, \lambda)\},$$

which means that O is certainly in a_1 or a_2 or a_3, and most likely in a_1 or a_2. Let f be given on the discretized space for the ai's. Note that $f(a_i \Delta a_j) = f(a_i) \Delta f(a_j)$. Here, let us assume that we have the information:

$$f(a_1) = a_2 \Delta a_3, \quad f(a_2) = a_3, \; f(a_3) = a_4, \text{ and } f(a_4)=a_2.$$

Note that f can be computed on the discretized space from its expression in the physical space offline.

We can check using the previous steps that $\sum'=f(\sum)$ is:

$$\sum' = \{(a_2 \Delta a_3 \Delta a_4, 1), (a_2 \Delta a_3, \lambda)\}.$$

Note that this syntactic treatment can be easily extended when f has a fuzzy image, e.g., $f(a_1) = \{(a_2 \Delta a_3, 1), (a_4, \alpha)\}$, taking advantages of the fact that $f(\Sigma)_{\varepsilon a} = f(\Sigma_{\varepsilon a})$.

Now computing $\sum_{t+1}$ from $\sum'$ is immediate using the previous subsection. Namely, let $x = Inc(\Sigma' \approx \{(A, 1)\})$. Then:

$$\sum_{t+1} = \{(\phi_i, a_i) : (\phi_i, a_i) \; \sum' \text{ and } a_i > x\} \approx \{(A, 1)\}.$$

Going back to the example, let $A = a1 \Delta a3 \Delta a4$ be the input information (which implicitely using mutually exclusiveness constraints, this means that O is not in a_2). Then $\sum$ is updated into: $\sum_{t+1} = \{(a_3 \Delta a_4, 1), (a_3, \lambda)\}$.

A syntactic counterpart to updating can be easily obtained in a similar way. See also [16] for an example.

7. Conclusion

This paper has presented a preliminary investigation of the idea of filtering in the qualitative setting of possibility theory and possibilistic logic setting. In spite of some similarities, filtering and updating have been contrasted. Their respective roles for situation assessment and for acknowledging the dynamics of the world are still to be better analyzed.

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