Qualitative Modelling of Linear Networks in Engineering Applications

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Abstract. There is a real need for electrical circuit modelling and analysis tools at an intermediate level between analogue and state-based simulators. Numerical tools do not capture the abstractions valued by engineers and logical/functional models do not maintain sufficient affinities with the electrical properties in a system. Qualitative models have had some notable successes in filling this gap but a number of difficulties pose a barrier to further progress.

We describe current issues and some key problems in qualitative analysis of steady-state systems. We then present a new qualitative circuit analysis method based on a many-valued resistance quantity space with an orders-of-magnitude relation. The results of this technique are shown to solve key problem cases. We argue that the new scheme opens up considerably more modelling possibilities and the method can be tailored to the application domain, giving powerful modelling options.

1 THE IMPORTANCE OF QUALITATIVE CONCEPTS IN ELECTRICAL SYSTEMS

There has been a long standing interest in the ECAD community to find an intermediate modelling level between the gate-level simulators and other analysis tools used in digital logic and the electrical analogue simulators that work at the transistor level. Circuit analysis often involves reasoning about *both* qualitative or state information and the fine details of selected electrical parameters. Ideally this should be performed in an integrated and coherent environment. There is also a real need for qualitative circuit properties to be captured and incorporated in the simulators and analysis tools used on the electrical systems found in the automotive and aerospace sectors. These concerns are closely related and arise because our existing powerful numerical analysis tools do not maintain sufficient affinities with the important abstractions valued by engineers [8].

1.1 The switch-level modelling approach

The QR community has apparently been unaware of the efforts by ECAD research engineers to build an intermediate grain-size model of electrical circuits. This work was directed at reconciling detailed numeric data with qualitative properties and state information. A particularly promising approach, called “Switch-Level Models”, was based on the framework of many-valued logic [3]. This showed remarkable similarities with recent qualitative work [6]. For current and resistance, however, switch-level models used a finite set of values giving a range of intermediate levels. The aim was to support a form of approximate numerical analysis by reducing the grain size downwards as appropriate. Unfortunately this formulation was seriously flawed and gave much worse accuracy than expected because of the error combination effects which involved operations like subtraction on imprecise values. Similar problems can be experienced with interval arithmetic. These limitations, which killed off this line of modelling, are described in detail in [1].

1.2 Qualitative circuit models

Independently, the QR community has developed qualitative electrical models and investigated their application to significant problems in design and analysis. In this paper we are concerned with steady-state rather than dynamic models. Such quiescent current models are usually based on a three-valued resistive mesh abstraction. The motivation for such work is to develop models that are more abstract and less complex than their numeric counterparts in order to capture the essence of circuit features and support reasoning processes that relate to the concepts and descriptions used by engineers.

In this paper we introduce a method that deals with some of the difficult remaining problems in this field. The next section describes important issues and examples of typical problems. We then describe a new circuit analysis method based on a many-valued resistance quantity space. We give results that show solutions to the presented problem cases and discuss their implications. Finally, we review related work and conclude that the new scheme opens up considerably more modelling possibilities and offers solutions in many application specific problem domains.

2 CURRENT PROBLEMS WITH QUALITATIVE CIRCUIT MODELS

Although considerable progress has been made there remain a number of difficulties that hinder qualitative circuit modelling methods. We are concerned with three important issues:

Structural and non-structural changes Most recent work on qualitative steady-state circuit modelling uses the electrical property of resistance as a first order approximation for any energy absorbing component. This gives a qualitative representation of resistance as a three-valued quantity space, $[0, +, \infty]$, corresponding to *short circuit*, *load* and *open circuit* respectively. These minimal models have been surprisingly effective partly because they abstract significant information about the circuit in an intuitive form; in contrast with the mathematical models and voluminous data produced by numerical simulation. However, the applications that use such models can only deal with changes that are structural in nature. This is because the quantity space corresponds to three distinct topological conditions: either nodes are electrically identical,

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or connected, or disconnected. If the specific requirements of the task can be satisfied with only structural changes, as is possible for FMEA (Failure Mode Effects Analysis) for example [5], then these models can suffice. But if we wish to analyze faults which cause only parameter changes, e.g. increase or decrease in a resistance value, then further development is necessary. This structural limitation applies to most qualitative circuit modelling [4].

Vivid models One of the attractions of the three-valued resistance model is its simplicity and directness. It retains a relationship with the graphical form of electrical circuits, unlike equations, and seems to exhibit a closeness with our conceptual basis of the real world. This directness of representation has been called vividness [7] and is a way of providing a simple conceptual structure. This seems important for human cognitive processes and helps maintain affinity with engineering concepts. However, extending the range of models to cover more reasoning tasks involves additional features that may impact on the accessibility and value of the model for human users. For example, the tree decomposition processes used by [9] to convert any resistive mesh into a single equivalent resistor are not easy to trace and understand. We must avoid such complications when we refine the coarseness of the minimal models and endeavour to retain vividness.

Generality and efficiency Some methods report a lack of generality in that certain circuit topologies can not be handled, for example, multiple-star circuits [4]. Many forms of unusual circuit configuration actually occur in practice [12] owing to the incremental construction of designs and the reuse and combination of fragments caused by the variant management task. A particular problem topology is the bridge circuit and this is discussed below. Regarding efficiency, the combinatorial complexity of some QR methods can be prohibitive for real world applications. We wish to build systems that can handle problems with, e.g., ten thousand components and therefore any algorithms developed must be of low complexity.

These are the factors we have taken into account when developing our method. We now list problems that frustrate all recent qualitative resistance modelling systems:

Bridge circuits A bridge is a significant kind of circuit topology that creates ambiguity in any qualitative model. Bridges occur when the ends of a circuit branch can not be resolved into an ordered potential difference and therefore the current magnitude and direction can not be determined. All circuits can be classified into those that can be reduced into a single equivalent value by repeated application of series parallel reduction rules (SP circuits) and those that can not be so reduced. Of the circuits containing bridges, some will be balanced, i.e. current magnitude = zero, and some will be unbalanced, with unknown direction of current flow. Because *it is impossible to deduce the state of current flow in bridges, for magnitude or sign, without exact quantitative values of the resistances and voltages involved*, this has been a major problem for all qualitative circuit models and no effective treatment of this difficulty has been reported.

Diodes and other uni-directional components Diodes are devices that act as a conductor in one direction but block flow in the other direction. Although diodes are used widely in DC circuits to control the selection of circuit functions and operations, they have not been given much attention by the qualitative modelling community. This may be because directional selectivity is a discontinuity that poses particular problems for models based solely on resistive elements. The FMEA system of [10] gets round this problem by performing multiple analyses, one for each possible direction of flow, and then selecting the result that matches the permitted flow direction of the diode. However, better solutions are desirable for more general applications.

Variable current levels Although a three-valued resistance model can cover a wide range of circuit components (to a first approximation) under steady-state conditions, this only allows a single current level to be represented: either current flows or it does not. There are occasions when an engineer needs to distinguish between different levels of current flow. For example, an electrically active motor might be running free (unloaded), running with normal load, or be in a stalled state. Each of these would give different current levels. It is possible to model a stalled motor as a short-circuit and this might be satisfactory for FMEA analysis, but other tasks might require to distinguish between the high load current from stalling and a complete short-circuit (that could have different effects). Similarly, an unloaded motor may take very little current, but still provide an active circuit path. In discussions with engineers we have discovered that three levels of current would provide a sufficient enhancement to cover a large range of applications. This translates into a five-valued quantity space for resistance which seems to offer a good match to engineers' intuitive models, at least as can be deduced from informal descriptions [2].

Low activity paths Another related problem concerns the distinction between fully active paths and paths with weak current flows. For example, a fault might cause a lamp to be fed from a high resistance source and therefore not actually function. Another case occurs for sneak circuit analysis where live but inactive branches give potential paths for faults in future operation. In a three-valued model all paths with *any* flow must be marked as active. It would be extremely helpful if any low-level paths could be marked separately. In automotive systems low-level but functionally inactive current flows are quite common.

3 A MANY-VALUED RESISTANCE MODEL

If we assume all electrical components can be replaced by suitable configurations of resistance, then all circuits become a resistive mesh represented by a graph E(T, R) containing T nodes and R weighted edges. We will assume two of the nodes are defined as the supply terminals. Instead of using the quantity space $[0, +, \infty]$ for the graph edges we extend the $+$ value to allow a series of different positive values corresponding to different strengths of connectivity. We have experimented with three such values which gives the quantity space: $[0, lo, med, hi, \infty]$.

Qualitative algebras for systems higher than three-valued are difficult and cumbersome [13]. Some of the operators are asymmetric and values can have mixed meanings, which makes for awkwardness and therefore less intuitive models. We believe the principles of many-valued logic can provide inspiration for higher level systems. However, we must be careful to avoid the multiple-level accuracy problems that ruined the switch-level modelling approach. Instead of allowing multiple levels for all variables we have investigated a many-valued resistance model.

It is very important to first establish the interpretation of any value system to be used. Consequently, we need to define the interpretation of $\{lo, med, hi\}$ and specify how they are to be resolved in series and parallel combinations. We define the ordinal relationship

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[2] We also note that most questionnaire designs, opinion data scores and many forms of psychometric testing use five point scales.

$0 < lo < med < hi < \infty$ to determine levels of restriction on current flow. The application semantics of these will be seen in examples below. The physics of series and parallel circuit reduction require the numeric summation of their resistances or their conductances respectively. Our qualitative version of this uses Max and Min as the series and parallel reduction rules respectively, (see [5] for more detail).

We define a chain as a path between two nodes (both of degree $\neq 2$), where all intermediate nodes have degree 2 and a segment is an SP reducible portion of a circuit. The analysis algorithm has three parts:

1. We locate all chains, calculate their equivalent resistance (using Max), replace them with a single edge and enter their details in a segment table. If a diode exists in a chain then the chain resistance is calculated and stored for both directions of flow. Next all parallel edges are located, replaced with their equivalent single edges (using Min) and entered in the segment table. This process is repeated for series/parallel in turn until either a single resistance or a non-SP reducible circuit remains.
2. We now apply a traversal algorithm to label the nodes in the circuit. This is based on the shortest path method of [5] and consists of two passes, from each supply terminal respectively. Paths of minimum segment resistance are followed and nodes are assigned values giving their resistance from each supply terminal. Following [5], two integers are also stored in each node and these give the minimum number of (segment) edges to the supply.
3. Finally, the qualitative values assigned to the processed graph are referred back to the segment table and values are calculated and assigned for each of the composite chains and branches in the original circuit. For a series segment, the maximum of the segment and the referred value is assigned to all edges and for a parallel segment, each edge is assigned the maximum of its own value and the referred value.

The final result is that each edge is assigned a resistance value that indicates (inversely) its level of current (or activity). Thus, a lo circuit edge might be shown to be carrying a current corresponding to a hi level of resistance. The example in figure 1 illustrates the method. Note that med is not shown explicitly but is a default, hence all edges except four have the value med. Following series-parallel reduction in stage 1 the circuit is simplified as in figure 2. Note that only one *non*-med value is now shown as the others have been absorbed by the reduction process. The node labels in figure 2 are the result of stage 2 — the number pairs, f/r, show the route taken on forward and reverse traversals. From these we see that all nodes are labelled med and only one segment has the value hi. The directions of flow between adjacent nodes are determined from the relative values of $f/(f+r)$ and this exposes the "reverse" flow across the bridge edge. Note that if these ratios had been equal the bridge would have been returned as "ambiguous". The final assignment of values to the original circuit is not shown but can be readily deduced by reflecting back the segment values into figure 1. Further examples follow.

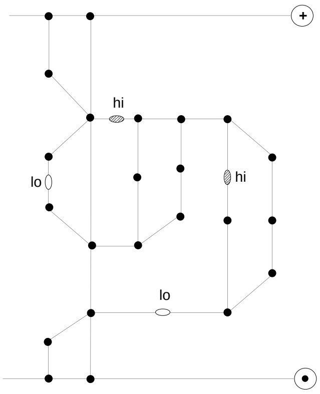

Figure 1. An example circuit

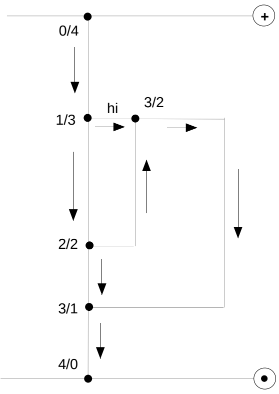

Figure 2. Path labelling scheme

4 RESULTS

We now present a series of case studies that show how the problems discussed in section 2 can be solved by the new model.

Diodes Figure 3 shows a circuit containing a diode arrangement that causes difficulties for previous models. When a diode is encountered its value is taken to be either 0 or hi depending upon the direction of flow. The correct direction (and segment resistance) is selected during the traversal. This produces the node labels seen in figure 3. We have designated hi as reverse diode impedance which is usually very high, consequently any branch containing a hi value has negligible current flow and so the active paths are clearly identified as the upper-left and lower-right diodes and the central load is powered. The meaning of hi is seen as being important in interpreting the results of any particular case. Note that we did not use ∞ for reverse diode resistance because it is useful to distinguish inactivity due to diode action from disconnection.

Bridge circuits A potential resistive bridge is seen in figure 4. Here all resistors are of value med except for the high impedance indicator and monitor device that are both hi. The results show the main flow path is through the three central resistors with two high resistance branches either side. The *relative* meaning of hi can be defined by the engineer who will also assess the significance of the lower current levels in the side branches. Without the many-valued distinction the central resistor would form a bridge that could not be resolved.

Variable current levels Figure 5 shows a motor circuit where the indicator changes from dim to bright according to whether the motor is powered. Because the indicator is linked to the motor, when the main switch is open there is a path for a small current flow through the motor. Previously we would have to record the motor as active, but now with a distinction between main power circuits

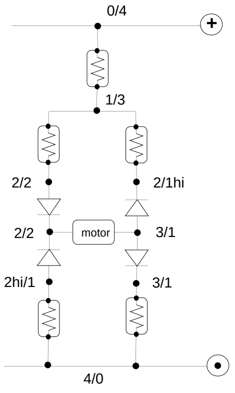

Figure 3. Diode regulator circuit

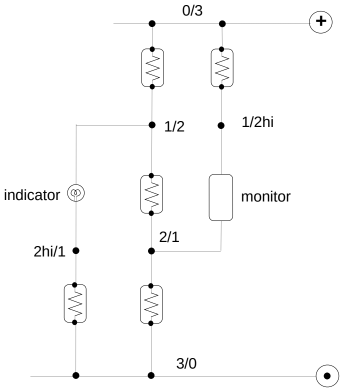

Figure 4. Bridge circuit

and signal circuits this can be resolved. The results show the motor to be on a high impedance path and this would be interpreted as being under-powered although electrically alive.

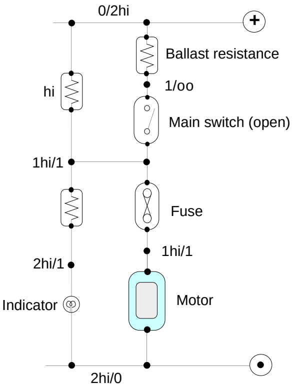

Figure 5. Motor drive circuit

Low activity paths Figure 6 shows an electronic monitoring module connected to a power circuit. It has been assumed that the input to the monitor is of high impedance and this has been represented by an internal resistive network, all of value hi. Note that the actual function of the monitoring module is not important — it may involve complex electronics or computing functions — all that is needed is an equivalent resistance model that holds for the external view of the circuit during the operation being modelled. The result is that all of the monitoring circuit has hi values and is distinguished from the power paths. Notice that previously this would have been another bridge circuit with ambiguous and unsatisfactory results.

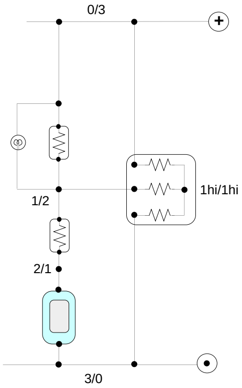

Figure 6. Circuit with voltage sensing device

5 DISCUSSION

It can be seen that a 5 valued quantity space, with 3 intermediate resistance values, has resolved previously ambiguous cases and opens up considerably more modelling possibilities. We have not resorted to Ohm's law to produce multi-valued current but have instead labelled each circuit edge (via the nodes) with a value that indicates (inversely) the maximum level of flow that can occur at that point. The interpretations given to the resistance values is a key feature of the method and these are determined by the application domain. In our examples we have seen the semantics of hi vary to distinguish: reverse diode leakage current, high impedance circuit sections, and low level signal currents. Others could include the increased impedance of an unloaded (free-running) motor.

We do not have space to include many examples of the use of the value lo but this value can be used to identify paths that carry higher current than "normal" and this offers more benefits for modelling key characteristics of problems. Similar examples demonstrate cases of lo resolving circuits with overloaded motors, partial shorts, and energy loss in conductors. This latter example is of special interest in automotive applications where all power cables that entail noticeable voltage-drops are to be monitored and distinguished from near ideal conductors such as signal wires.

Orders-of-magnitude relationships [11] have already been shown to be capable of solving some unbalanced bridge circuits [6]. We can see that such a relationship is contained within the series/parallel reduction rules because of the nature of the qualitative ordering. Hence, any number of med edges in series will always be considered lower in value than a single hi edge, and similarly with any other pair, according to the ordering. The Min function has the equivalent effect on parallel circuits. This means the ordering implemented is actually $0 << lo << med << hi << \infty$.

It is clear that, in general, bridges can not be solved by *any* qualitative method. If the four resistances surrounding a bridge are a, b, c and d, then balance is achieved if: $a * d = b * c$. This requires precise values and the best that qualitative methods can do is return the label ‘ambiguous’. However, bridge edges *can* be resolved when they are unbalanced and have order of magnitude differences in the relevant branches. Our formulation is an efficient way of discerning such cases that are definitely unbalanced.

6 RELATED WORK

An early contribution on linear steady-state systems was the work of [15]. Relevant recent work on three-valued quantity spaces includes [14] for applications in diagnosis and FMEA, [9] on the use of series/parallel/star/delta replacement rules to convert any qualitative resistive mesh into a single equivalent resistor, and [2] who applied similar series/parallel decompositions to analyze alternating current circuits. Another qualitative method described in [5] has been used by [10] to develop a successful full-scale commercial FMEA software tool that is now in regular use in the automotive industry.

The work of [4] is particularly relevant and describes the differences between existing (structural) methods and the requirements for analyzing non-structural changes. They emphasize the importance of deviations from a norm and develop a method using qualitative deviations. A set of circuit rules are used to propagate current and voltage values and a form of series-parallel-star tree reduction [9] is used to reduce the resistive nets. The main problems with this method are that it relies on topology dependent rules, it can not handle bridges, and it is not general for all circuit topologies. Like [9] this method can carry out numerical reduction in parallel but it suffers from a loss of vividness due to the complications involved in processing tree transformations. Other work on qualitative deviations includes [14].

Our method can also represent deviations: med can be a norm value and then hi or lo become variations, possibly due to a faulty component. For other tasks like FMEA we can compare two circuit analyses to locate the branches that have changed status; due to either structural *or* parameter changes. We see our method as a way of classifying the branches of a circuit, and the significance of the resulting branch labels will depend upon the initial application semantics given to the resistance value set.

7 CONCLUSIONS

We have presented a new qualitative circuit analysis method. A number of problems for previous systems, with both technical and application urgency, have been solved by the new model. These include diode circuits, variable current levels and unbalanced bridge circuits. The model has retained a high level of vividness with respect to the real world system; a feature considered important for acceptance and affinity with practicing engineers.

The model illustrates how larger quantity spaces for resistance can effectively label circuit nodes with application specific classes for different types of current flow. We have used three values of resistance but it is clear that increasing the quantity space set will deliver even more flow classes, representing increasingly finer resolution.

By not attempting the calculation of any current values we have avoided the combinatorial and accuracy problems that arise from arithmetic on several multi-valued qualitative variables as experienced in the switch-level models of [1]. However, the interpretation of the value set has been emphasized. We remember in Lukasiewicz’s 3-valued system, the semantics of the intermediate truth value was taken to mean *indeterminate*, while the 3-valued systems of Kleene and Bochvar interpret this value as *undecided* and *meaningless*, respectively. In the same way, the resistance semantics defined by the application are reflected in the resulting branch classifications obtained. This offers both modelling flexibility and coherence.

The method relies on an orders-of-magnitude resistance relation and requires a special path traversal algorithm to perform the labelling process. The method is efficient, $O(N^2)$ for N nodes, and is also general in that all topologies can be processed. The mapping between the reduced and original circuits is readily accessible from the segment tables and star/delta transformation rules are not required.

In summary, the method is general, robust, efficient and intuitive. It is flexible in that the qualitative value range is tailored to individual applications. The main limitation is that all qualitative values must be constrained by an orders-of-magnitude relation.

Our method can be viewed as a process that classifies circuit branches into different impedance levels. It might seem that we are still some way from ECAD systems that can reconcile numeric models with relevant qualitative circuit information. However, we suggest that, rather than aiming for total integration through convergence of representations, the operation of an analog circuit simulator in tandem with our qualitative model would produce very useful output. Such parallel operation could label each wire, node or component with both actual voltage/current and qualitative labels that signal important application specific conditions. This seems to offer considerable potential for a coherent modelling environment on which to build future engineering reasoning tools.

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