Emergence of Complex Networks through Local Optimization

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Abstract. The emergence of complex network structures of relationships between autonomous agents occurs in a wide range of distributed systems. Many researchers have proposed models to explain and reproduce this phenomenon. However, the mechanisms proposed require implausible assumptions of global knowledge about structural positions, and do not make explicit individual agents' goals that motivate decisions to establish or break ties. We propose a model grounded in social exchange theory that is based on a population of agents with dissimilar attractiveness levels, who seek to optimize the set of their exchange partners in terms of the outcome they receive in a support game. In our model, agents perform a local optimization process constrained by realistic and plausible assumptions of local and imperfect information. We show that our model can generate several types of complex networks, notably power-law, small-world and center-periphery networks. This diversity of network structures is related to a set of parameters that shape the *harshness* of the exchange problem.

1 COMPLEX NETWORK EMERGENCE

A wide variety of distributed systems composed by autonomous agents strikingly display the same particular topology on their network of interactions, despite the fact that there is no engineered architecture whatsoever. Some examples are the Web, communication networks, social networks, ecological graphs, sex contacts networks, scientific collaboration, or terrorist networks. These kind of networks, called *complex networks*, turn out to be very efficient in terms of information propagation [12, 8], are highly robust against random errors [1], and emerge from the interactions of numerous agents' who act motivated by their particular individual goals.

Researchers in many fields have proposed models to reproduce the emergence of complex networks, such as small-world and power-law (scale-free) structures. Initially, the seminal work by Watts and Strogatz proposed the mechanism of *stochastic rewiring* [14]. Subsequently, the well-known Barabási mechanism of *preferential attachment and uniform growth* [3] was introduced, which then has inspired many of the now existent models in the literature about generation of power-law networks. For a complete summary see [2]. Models proposed in this literature rely on public and full access to reliable knowledge about the whole network. For instance, they assume that an agent can know the connectivity degree of any other agent of the system. This assumption is clearly unrealistic for many real world networks. However, the assumption is needed to obtain a parametric and analytical solution that can reproduce real networks observed in nature with high reliability. Thus, these models *reproduce* rather than *explain* complex networks. Several researchers have addressed this issue [9, 5]. They proposed a *transitive linking* mechanism that is solely based on agents' local ties. This resulted in a model of *preferential attachment* behaviour without global knowledge. However, this class of models, as well as the original stochastic rewiring and preferential attachment models, do not explicate the goals and cognitions that motivate agents' decisions to make or break ties. In our opinion, modelling plausible individual level decision mechanisms is the key for understanding the emergence of such structures. This view is recently also advocated by Robins et al [11], who propose a class of models where local interaction rules specified in a Markov processes can generate small-world structures. We refer in the following to systems where agents have goals and some sort of cognitive capabilities, as is the case for multi-agents or human social systems.

A further approach, without *preferential attachment* as baseline, is to model the emergence of complex networks as result of an *optimization process*. For example, Carlson and Doyle [4], propose the approach known as Highly Optimized Tolerance (HOT), where power-law networks emerge as a response to a certain internal hazard, resembling a natural selection process. Another model based on optimization is by Ferrer and Solé [6]. Here, agents perform optimization of global structural measures of the network, such as density and average path length. This model similarly uses implausible assumptions about individual agents' knowledge of the whole system and perfect information.

We propose a model that follows the optimization approach, but is based on local information and *local optimization* driven by individual agents' goals. The goals are made explicit in our assumptions. Notice that this approach is not incompatible at all with the *preferential attachment* model. In fact, agents who optimize their individual networks can also display preferential attachment. However, when agents make or break ties, this is seen a consequence of their individual optimization process. For example, in scientific collaboration, scientists do not choose their co-authors in terms of who is the most connected, but in terms of who is the best to conduct research with. At the same time, it likely that the most connected agent (scientist) is also the most attractive one [10]. In this example, preferential attachment is not the underlying mechanism, but it is a consequence of the local optimization process.

2 LOCAL OPTIMIZATION

Our model is grounded in social exchange theory and assumptions of bounded rationality. More specifically, we draw on previous work by Flache and Hegselmann [7] and assume that agents seek to find

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and keep attractive exchange partners in a population where agents differ in attractiveness. They could, for example, differ in their degree of expertise in knowledge exchange. In that way, outcomes from knowledge exchange with their partners is maximized.

Agents pursue their goals by maximizing outcomes from support exchange under imperfect, local information without initial knowledge about others or knowledge about the global network structure. Moreover, agents in our model are strongly adaptive, i.e. they acquire knowledge only acquired in the course of interaction and simple search heuristics are applied. However, as we will show, these assumptions do not prevent the emergence of complex networks.

2.1 Model Description

The system consists of a population of N agents, who are each endowed with an individual attractiveness value, α_i, that is a real value in the $[0.05..0.95]$ range. Initially, every individual agent is ignorant of every other agents' attractiveness, even of its own. This assumption introduces the typical uncertainty of distributed systems into our model. Attractiveness levels are initialized randomly based on a uniform distribution. Agents are also endowed with a memory: $M_i^t = \{(o_{ij}, t_{ij})\}_{j \in J_i^t}$, where o_{ij} is the expected outcome received by a_i after interacting with a_j at time t_{ij}. Each agent a_i can only manage a part of the whole system: $J_i^t \subseteq \{1..N\}$. This part is limited $\#J_i^t \leq M_c \forall t \forall i$. Thus agents only have access to a partial view of the whole community, and their knowledge exclusively depends on their past experience. Therefore, the knowledge of agents remains local. Within the set of agents that a_i can manage, there is a partition into *known* agents K_i^t, and *unknown* agents U_i^t, such that $J_i^t = U_i^t \cup K_i^t$ and $U_i^t \cap K_i^t = \emptyset$. Initially, each agent is initialized with M_o agents chosen randomly that are classified into the *unknown* set. That is, a_i knows that those agents exist but knows nothing about them. All other agents are not only unknown for a_i, but a_i is not even aware that they exist. Throughout the simulation, a_i will learn about those agents and they will be moved to the set of *known* agents; K_i^t. Accordingly, the outcome that a_i expects to receive from interaction with a_j is only defined when $j \in K_i^t$. All the sets and memory of a_i depend on time t since the memory and sets evolve after interactions. Moreover, memory, and consequently the U_i^t and K_i^t sets, are always bounded by the maximum memory capacity M_c.

The system runs for S units of time. In one time unit, all agents execute their action rules once, activated in random sequence. Once activated, the agent tries to establish Q interactions with agents contained in its memory such that it maximizes the expected outcomes from those interactions. Interactions are dyadic and both participants need to agree before they carry out an interaction. Accordingly, we introduce a limitation on the number of concurrent interactions, which we call *capacity* (C). Capacity is the maximum number of interactions per time unit. Capacity must be equal to or smaller than the memory capacity, $C \leq M_c$. For example, when an interaction implies a certain *work load*, it is implausible that agents have unlimited capacity. Yet, this is another assumption taken for granted in most of the models in the literature. However, when an interaction does not imply a work load for both parts, such as deploying a link from one web page to another, capacity is unrestricted. But interactions in most social processes usually require allocation of certain resources, which are, indeed, limited.

2.2 Support Game Definition

Outcomes of an interaction are defined in the payoff matrix of the support exchange game, P.

Table 1. Payoff Matrix of the Support Exchange Game P

P	a_j *collab (C)*	a_j *defect (D)*
a_i *collab (C)*	$(G_{ij} - L_{ij}, G_{ji} - L_{ji})$	$(-L_{ij}, G_{ji})$
a_i *defect (D)*	$(G_{ij}, -L_{ji})$	(0, 0)

Where,

$$G_{ij} = (1 - \alpha_i)\alpha_j B \quad (1)$$

$$L_{ij} = \alpha_i(1 - \alpha_j)E \quad (2)$$

The payoffs both players receive after an interaction, p_{ij} and p_{ji} depend on the attractiveness of both agents (α_i and α_j), the benefit parameter B, and the effort parameter E. The parameters *benefit* B, and *effort* E, such that $E < B$, are positive constants that weigh the benefit of receiving one unit of help against the effort costs of providing the unit.

The payoff received is not symmetric unless agents have equal attractiveness. The gain of the interaction, specified in equation 1, increases when a_i interacts with a partner a_j who is more attractive, for example because the partner has more expertise. The loss from the interaction, specified in equation 2 , increases to the extent that a_i interacts with an a_j who is less attractive. Broadly speaking, *good* agents want to interact with *better* agents. The bigger the attractiveness gap is, the more interesting it is for the less attractive agent. Conversely, the less interesting the interaction is for the more attractive agent. For example, when we seek advice on a given topic, our profit is maximized if we can ask a highly knowledgeable person, a *guru*, but the guru receives little new knowledge from us.

It is important to remark that the constituent support game is a PD *(prisoner's dilemma)* if, and only if, for both players it holds that $(1-\alpha_i)\alpha_j B > \alpha_i(1-\alpha_j)E$. In a PD, cooperation is difficult to attain, because both players may profit from exploiting their partner, while none of them would prefer mutual defection to mutual cooperation. However, the support game is not always a PD. In fact, the game can be purely collaborative with no incentive to defect when $E = 0$. In that case, interactions are then always profitable for both parts. Yet, they are not symmetrical, because an agent might receive a better payoff than its partner.

Due to the local, imperfect knowledge assumption of our model, the parameters B and E, as well as the attractiveness levels are not explicitly known by the agents, who thus pursue their goals facing a high degree of uncertainty. The only knowledge an agent has is: the outcome obtained after an interaction, p_{ij}, the *net benefit* $(G_{ij} - Lij)$ of an interaction (explained in the next subsection), and the initial set of agents U_i^t about whom he knows nothing but their existence.

2.3 Agents Behaviour

When an agent a_i is activated, it behaves as follows. Agent a_i selects another agent a_k to whom it proposes an interaction at time t.

$$a_k = \begin{cases} k \in J_i^t \text{ chosen at random} & \text{with prob. } e_i^t \\ k = argmax_{n \in J_i^t}(o_{in}) & \text{with prob. } (1 - e_i^t) \end{cases} \quad (3)$$

With exploration probability e_i^t an agent within a_i's memory is chosen randomly. Otherwise, the agent with the best expected outcome is chosen. The probability for exploration is defined by the following equation.

$$e_i^t = \frac{\#U_i^t}{(\#J_i^t)^2} \quad (4)$$

The exploration probability is 1 at the beginning of the simulation and tends to 0 as a_i as agents get to know other agents. Accordingly, the probability soon varies between agents. Very successful agents quickly decrease their exploration probability, because they receive a lot of interaction proposals, and, therefore, learn about their environment faster than agents who receive less interaction proposals. Then, the chosen agent a_j must decide whether to accept the proposal of agent a_i. The proposal will be accepted if a_j has not reached its capacity C of concurrent interactions, and a_j expects the outcome to be not negative, given a_i is known by a_j.

$$accept_{ji} = (\#(a_n \in J_j^t | t_{jn} = t) < C) \wedge (a_i \in J_j^t \Rightarrow o_{ji} \geq 0) \quad (5)$$

If the interaction proposal is accepted, both agents play the support game defined by the payoff matrix, P. In this version of our model, we assume that agents are *benevolent*. Agents are not aiming to exploit their partners, but they may still defect. An agent a_i collaborates with a_j if, and only if the *net benefit* is positive, $(G_{ij} - L_{ij}) \geq 0$. Once agents decide to interact, this value is given by the system to the agents, because they cannot calculate the *net benefit* by themselves due to their limited knowledge. The payoff information is explicitly hidden in the model in order to create a realistic counterpart of a distributed, open system. The step of giving the *net benefit* can be justified in terms of each agent *cognitive capabilities*, which allow an agent to assess at an early stage of an interaction whether it will be profitable or not.

After an interaction is completed, agents update their respective memories. If the entry for the interaction partner does not exist, agent must allocate a free slot in their memories. When the memory is full, a slot from the memory is selected for replacement of the agent a_k that it currently represents, according to equation 6.

$$a_k = argmin_{n \in J_i^t}(| \, o_{in} \, | \, |(t_{ik} < t)) \quad (6)$$

The memory is updated depending on whether a previous interaction between both agents already exists or not, as shown in the next equation.

$$(o_{ij}, t_{ij}) = \begin{cases} (\frac{o_{ij}+p_{ij}}{2}, t) & \text{if, } t_{ij} > 0 \\ (p_{ij}, t) & \text{if, } t_{ij} = 0 \end{cases} \quad (7)$$

Where t refers to the current simulation time, p_{ij} denotes the payoff attained by i after an interaction with j, and t_{ij} refers to the time when the last interaction between i and j took place. When an interaction proposal is rejected the payoff p_{ij} is set to zero.

Agents exchange some knowledge about their respective memories as an act of deference after a mutual profitable support interaction, when $p_{ij} > 0$ and $p_{ji} > 0$. We distinguish two types of knowledge exchange, *explicit* and *implicit*. In explicit exchanges, a_i informs a_j about the third agent in the memory of a_i with the best expected outcome. This can be seen as *referral* to another agent. More concretely, the referred agent is chosen according to $a_k = argmax_n(| \, o_{in} \, | \, |(k <> j \wedge t_{ik} > 0))$. In implicit exchange, a_i picks a randomly chosen agent with whom a_j currently interacts, from a_j's. With *implicit* exchange agents learn through observation of each others' environment.

The new knowledge acquired by memory exchange $(o_{ik}, t_{ik}) = (o_{jk}, 0)$ is handled like in the updating process for knowledge obtained from own experience. Notice that a_i provides its subjective expected outcome about the third agent, o_{ik}, to a_j. Agents do not have the capacity to assess which third party is more suitable for their partner a_j. Time t_{ik} is set to 0 in the updating, to reflect that a_i's knowledge about a_k stems from referral or observation rather than experience.

3 RESULTS

Our model has a large number of parameters. We focus here on simulation experiments for which a number of the model's parameters are fixed to values that are plausible within the framework of social exchange processes. The capacity of agent's memory, M_c, is set to 200, the number of concurrent interactions, C , is set to 150, and the set of initial agents, M_o is also set to 150. We conducted several series of simulations. We modified the population size, N, the number of interaction proposals Q, the type of memory exchange, (henceforth ME) and the type of the support game represented in terms of the ratio between effort and benefit, $\frac{E}{B}$. In order to compare the results generated by our LO-model (Local Optimization Model) with those of other models that are known to generate complex networks, we used the model of Walsh [13]. This WN-model is based on *preferential attachment and uniform growth*, and assumes total knowledge about the whole system structure, like most of the models of that family. LO-model networks are based on the interaction networks that emerged after 100 units of simulation time. Notice that the exploration probabily as specified in equation 4 assures the convergence to an stable state, since exploration probability diminishes over time. At convergence, an undirected interaction-edge (i, j) indicates that a_i and a_j have a stable mutual support relation.

Table 2. General characteristics of simulated networks. LO (Local Optimization) denotes our local optimization model. WN refers to the Walsh model. Q, which corresponds to the average degree $\langle k \rangle$, is set to 5. The population size, N, is set to $\{1000, 5000, 10000\}$, the cost to benefit ratio ($\frac{E}{B}$) is set 0, $\frac{3}{16}$, $\frac{8}{16}$, and the memory exchange is set to $\{E, I\}$ i.e. explicit and implicit respectively. For each network, the average path length l and the clustering coefficient C are compared to the average path length l_{rand} and clustering coefficient C_{rand} of a random graph with the same size and average degree.

Network	*Size*	l	l_{rand}	C	C_{rand}
$WM_{(m_o=150)}$	1000	3.30	3.27	0.016	0.0085
$LO_{(E/B=0,I)}$	1000	2.47	3.27	0.16	0.0085
$LO_{(E/B=0,E)}$	1000	2.51	3.27	0.20	0.0085
$LO_{(E/B=3/16,I)}$	1000	3.66	3.27	0.052	0.0085
$LO_{(E/B=3/16,E)}$	1000	3.71	3.27	0.034	0.0085
$LO_{(E/B=8/16,I)}$	1000	3.98	3.27	0.012	0.0085
$LO_{(E/B=8/16,E)}$	1000	3.91	3.27	0.027	0.0085
$WM_{(m_o=150)}$	5000	3.67	3.97	0.005	0.0016
$LO_{(E/B=0,I)}$	5000	3.22	3.97	0.05	0.0016
$LO_{(E/B=0,E)}$	5000	3.16	3.97	0.085	0.0016
$LO_{(E/B=3/16,I)}$	5000	4.09	3.97	0.038	0.0016
$LO_{(E/B=3/16,E)}$	5000	4.14	3.97	0.030	0.0016
$LO_{(E/B=8/16,I)}$	5000	4.45	3.97	0.015	0.0016
$LO_{(E/B=8/16,E)}$	5000	4.57	3.97	0.040	0.0016
$WM_{(m_o=150)}$	10000	3.87	4.30	0.003	0.001
$LO_{(E/B=0,I)}$	10000	3.56	4.30	0.034	0.0085
$LO_{(E/B=0,E)}$	10000	3.42	4.30	0.065	0.0085
$LO_{(E/B=3/16,I)}$	10000	4.29	4.30	0.032	0.0085
$LO_{(E/B=3/16,E)}$	10000	4.37	4.30	0.029	0.0085
$LO_{(E/B=8/16,I)}$	10000	4.68	4.30	0.017	0.0085
$LO_{(E/B=8/16,E)}$	10000	4.84	4.30	0.038	0.0085

Table 2 shows that the networks obtained from the LO model have a similar average path length compared to random graphs of the same size and average connectivity, $l_{LO} \simeq l_{rand}$. Moreover, we see that l_{LO} scales logarithmically with N, as l_{rand} does. Finally, the clustering coefficient of LO is much higher than that from random graphs, $C_{LO} \gg C_{rand}$. Taken together, our simulated networks thus exhibit the small-world properties defined by Watts and Strogatz [14]. We conclude that the networks generated by LO have indeed the *small world* property. The table shows that the networks also comply with the criterion proposed by Walsh [12]. This criterion is that the network can be classified as small-world, if $\frac{l_{rand}C}{lC_{rand}} \gg 1$. Of course, the network generated by the Walsh model is also small-world, because it is a model that generates power-law networks. However, when we compare the Walsh and the local optimization model, we see an important difference in the clustering coefficients, $C_{LO} \gg C_{WN}$. It seems our model well resembles the high clustering coefficients that the literature reports for many real complex networks, especially social networks [2]. At the same time, many models proposed in the literature partially fail to reproduce clustering levels this high.

Which type of a small-world network we observe can be seen in the connectivity distribution. In figure 1 we show the connectivity distribution of networks produced with the LO model. The figure shows that different kinds of connectivity distributions arise for different parameter settings. We can identify three clearly distinct types:

1. star-like distribution (figures 1.a1, 1.a2): while this distribution does not correspond to a perfect star network, the underlying network is similar to a star with its *center-periphery structure*. In this structure, a set of nodes with high clustering among the set becomes the core of the network, and the majority of nodes connects solely to the core. Networks of this type have small-world properties, despite their resemblance to star networks.
2. potential distribution (figures 1.b1, 1.c1, 1.c2): $P(k) \sim k^{\gamma}$, is the signature of the degree distribution of a *power-law network*, also called *scale-free network*. This is the paradigmatic complex network structure [2]. Most of the literature focuses on power-law networks.
3. exponential distribution (figures 1.d2, 1.e1, 1.e2): The degree distribution exponential networks is given by $P(k) \sim \theta^{-k}$. Exponential networks are known to have similar properties than small-world networks. Their main difference to power law networks is the steeper decline of the degree frequency as degree increases.

The results show that the LO model can not only generate power-laws, as many of the models in the literature also do, but also center-periphery networks and small-world networks with exponential connectivity distribution. For this, we only need plausible assumptions about agent's cognitive abilities, uncertainty and access to information. At the same time, figure 1 demonstrates that a variety of structures can be generated by the same behavioural model. When the cost to benefit ratio $\frac{E}{B}$ is changed explicitly, this has a strong effect on the macro structures that emerge from the agent level interactions. The ratio $\frac{E}{B}$ has a direct effect on the *system harshness*, which we define informally as the difficulty for agents to attain their goals. With $\frac{E}{B} = 0$, the system is collaborative. Accordingly, it is easy for an agent to find a profitable set of partners in spite of the existing uncertainty. However, when $\frac{E}{B}$ tends to 1, the system progressively becomes more exigent and competitive, making agents' task more difficult.

Our experiments show that the cost to benefit ratio is not the only factor that affects the *system harshness*. Any parameter or constraint that affects the *system harshness* also affects the type of macro-structure that emerges. For example, our results indicate that the type of memory exchange has an effect on the macro structures, as shown in figure 2. This figure summarizes the results we obtained from different parameter settings, in order to demonstrate how the memory

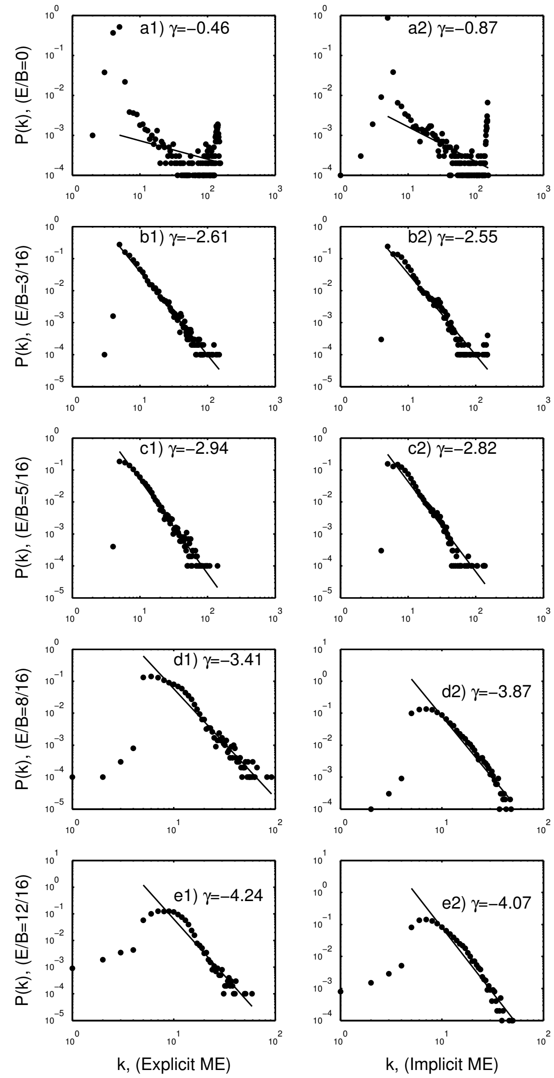

Figure 1. Connectivity distribution obtained from the Local Optimization model. Q, or $\langle k \rangle$ are set to5, N is set to 10000, in the left column the memory exchange is explicit, in the right column it is implicit. The cost to benefit ratio, $\frac{E}{B}$ is set to $\{0, \frac{3}{16}, \frac{5}{16}, \frac{8}{16}, \frac{12}{16}\}$.

exchange affects the final structure.

Figure 2 can not show connectivity distributions. Accordingly, we rely on the determination coefficient of the potential regression r^2. This coefficient has unfortunately some drawbacks. For example, the effect of the population size N on r^2 is misleading, due to the scarcity of regression points when $N = 1000$. Another problem is the high value for r^2 in the *exponential distribution (ED) space* with explicit memory exchange. An exponential regression determination coefficient fits much better with those distributions. Figure 1.e2 illustrates this issue. The connectivity distribution in this figure is clearly exponential. While its potential r^2 is high, its exponential r^2 is even higher with 0.98. In cases like this one, we classify the distribution as exponential rather than potential. Notice that the regression is carried out over all points except those where $k \leq Q$, as it is usual in complex-networks analysis since the connectivity baseline is Q. For the sake of clarity we do not display both potential and exponential determination coefficients.

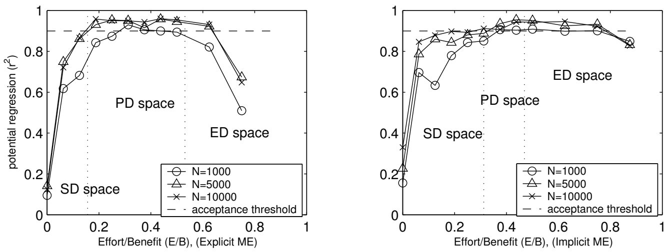

Figure 2. Comparison of the potential regression determination coefficient r^2 for network generated by LO. The left figure corresponds to explicit memory exchange, and the right one to implicit. For each subfigure the population size, N, set to $\{1000, 5000, 10000\}$, and the cost to benefit ratio, $\frac{E}{B}$ is set to $\{0, \frac{1}{16}, \frac{2}{16}, \frac{3}{16}, \frac{4}{16}, \frac{5}{16}, \frac{6}{16}, \frac{7}{16}, \frac{8}{16}, \frac{10}{16}, \frac{12}{16}, \frac{14}{16}\}$. We use as acceptance threshold $r^2 = 0.9$. The vertical lines separate the different regions in the parameter space: SD (star-like distribution), PD (potential distribution, power-law), ED (exponential distribution).

Figure 2 shows how the type of memory exchange modifies the boundaries of the *star-like, potential and exponential distribution spaces*. We do not observe a phase transition between those spaces, the transition is progressive and the boundaries are fuzzy.

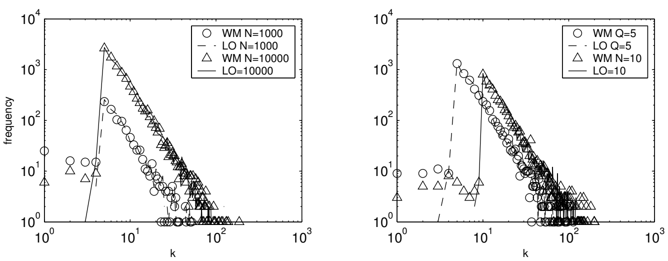

Figure 3. Comparison of connectivity distribution. LO is the local optimization and WM is the Walsh model. N is set to $\{1000, 10000\}$, Q is set to $\{5, 10\}$, $\frac{E}{B}$ is set to $\frac{3}{16}$, the memory exchange is implicit. Parameters of the Walsh model are: m set to $\{5, 10\}$, and m_o set to 150, corresponding to the parameters Q and M_o of the LO model, respectively. Symbols represent the Walsh model, solid lines pertain to the LO model.

We focus on two concrete examples to illustrate this effect. With $\frac{E}{B} = \frac{3}{16}$ and implicit memory exchange, the system is not harsh. Accordingly, the emergent structure is a center-periphery network. However, with explicit memory exchange the system becomes harsher, because the most attractive agents are known for a large portion of the population. Hence, these most attractive agents enter in what we call an *on-denial stage*, which is produced when an agent receives more interaction proposals than it can handle due to the capacity limit C. Then, it must decline some interaction proposals. Consequently, the agents that get rejected reduce their expected outcome on that agent, diminishing its subjective attractiveness in turn. The *on-denial* stage introduces noise into the system and thus increase the complexity of agents' problem solving task. This, in turn, affects the global structure such that it shifts into the *power-law space*. At the same time, when $\frac{E}{B} = \frac{8}{16}$ and the memory exchange is explicit, the system is is moderately harsh. In this case, it is in the *power-law space*. However, with implicit memory exchange, learning is exclusively based on the observation of raw knowledge. Learning becomes more unlikely to be valuable as the uncertainty of the system grows. Consequently, the system gets harsher and shifts into the *exponential space*.

We also studied how the system scales, when the population size N and the number of interaction proposals Q change. The effect of the population size has already been shown in figures 1 and 2. We now compare the obtained connectivity distribution from the local optimization model with the one obtained from the Walsh model [13]. Figure 3 demonstrates that the LO network scales similarly to the Walsh network in response to modification of N and of Q. Moreover, both networks are almost identical, which supports our claim that the networks generated by the LO model correspond well to power law structures.

4 CONCLUSION

We showed that complex networks can emerge from an agent level model that assumes local optimization over individual outcomes that agents receive from bilateral support exchanges. Our model assumes a high degree of uncertainty for the agents due to local and imperfect knowledge assumptions. Furthermore, we have shown that the type of complex network obtained depends on properties of the system itself, such as the *system's harshness*. Changes in the qualitative macro patterns can thus be obtained without changes in the underlying agent level behavioural model. System harshness is modified through parameters of the support game or the memory exchange. These modifications allow to generate different types of complex networks, including the celebrated power-law networks. We conclude that our local interaction model provides a plausible explanation of how complex networks may emerge, at least for the context of social exchange processes.

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