Expectation Reasoning using Regret and Disappointment

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Abstract. A critical aspect of an agent system is the ability to deal with unexpected situations to determine an appropriate course of action in a changing environment. In this paper, we investigate the incorporation of the mental attitudes of regret and disappointment (which have been studied by economists using utility theory) into the agent's reasoning system in order to improve its ability to deal with unexpected events. Mental attitudes in agent systems have generally been expressed in modal logics, such as the Belief-Desire-Intention (BDI) logic and epistemic logic, and, more recently, in a logic of expectation and observation. We show how regret and disappointment can be naturally integrated into a framework based on the attitudes of expectation and observation, and describe some key properties of the system.

1 INTRODUCTION

Every day, we deal with many observations that are out of our expectations. We, then need to make some decision to adjust our behaviours according to such changes. Our software agents also constantly face such situations. Imagine, NASA's Mars Exploration Rover Spirit, suddenly, starts having an overloaded problem with its flash memory due to a large amount of requests from the land controller. It has a number of options: Stop executing operational instructions, wait for all data being retrieved and cleared up; keep collecting data as requested until memory is totally full; send all self-examination data back to the land controller, etc. Research in economics such as Bell [3, 4] and Loomes-Sugden [14] reveals that, if a person were put in such situation, one's decision would also depend upon how that person would feel when comparing the decision consequences together and with prior expectations. The negative difference between expectations and actual observations usually results in some mental states which would strongly effect the person's future decision. Such mental attitudes are ***regret*** and ***disappointment***.

In recent years, the concept of agent has become increasingly popular. The term agent represents a new synthesis for a variety of subdisciplines in AI and, more generally, computer science. An agent is recognised through the characterization of action that it can take to meet its design objectives in its situated environment. Those actions are usually required to be autonomous, flexible (proactive, reactive), and cooperative [29]. Such capabilities make agent become highly suitable for applications which are embedded in complex dynamic environments. Agent technology has been used in areas for applications such as air traffic control, automated manufacturing or even space operations like the above Mars rover example.

Mental attitudes such as knowledge, belief, desires, intentions, etc. have been formally analysed to predict the intelligent behaviour of an agent following a dominant approach by Dennett [8], "agents as intentional systems". Since the seminal work by Hintikka [11], such studies are usually carried out using modal logics with possible-worlds semantics. Among these models, Belief-Desire-Intention (BDI) model [6] and BDI logics [20] have been one of the most successful. Unfortunately, BDI logics are usually claimed as having *ungrounded semantics* [30], that is, there was no work showing a **one-to-one correspondence** between a mental model and any concrete computational interpretation. A recent work by Trần et al [26, 25] has offered a more general framework to overcome this problem. In this work, the central concepts of expectations and observations are formalised to realise Popper's logic of scientific discovery [16, 17, 18]. Apart from the sensory observations that an agent can experience from its environment, the framework also recognises the importance of effective observations as devices which produce uncertain perceptions to an agent. The uncertainty is cleared up when expectations of effective observations are justified by sensory observations.

In this paper, based on the justification of effective observations by sensory observations, we describe the mental attitudes regret and disappointment in expectation logic. Section 2 briefly introduces the approach by Bell [3, 4] and Loomes and Sugden [14], who integrate the concepts of regret and disappointment into utility theory. In section 3, we take a different approach to represent these attitudes using expectation and observation framework. Section 4 describes properties of observation system to deal with unexpected events based on regret/disappointment averse attitudes.

2 RISK, DISAPPOINTMENT AND REGRET THEORY

Uncertainty is a fundamental property of many agent environments. There have been different classification of uncertainty. Frank Knight [13] distinguished uncertainties in two sorts: "*measurable uncertainty*" (or *risk*), which may be represented by numerical probabilities and "*unmeasurable uncertainty*", which cannot. Knight also maintained that although it is impossible to assign numerical probabilities in the latter "uncertainty", it prevailed. Theories, that leave out the latter, would suffer certain violations in explaining human behaviour. The inappropriate use of von Neumann and Morgenstern [28], Savage [21] axioms have been shown by many other researchers such as Allais [1], Kahneman and Tversky [12], Bell [3, 4], and Loomes and Sugden [14].

In such works, there exists a common assumption that even in situations involved unmeasurable uncertainty, people tend to behave "as though" they assigned numerical probabilities. In 1982, Bell [3] and Loomes-Sugden [14] simultaneously presented the regret-rejoice model. Although there is a slight difference between two works, they introduce a regret (disappointment) function to represent the psychological state when comparing consequences of their actions. In re-

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gret theory [3, 14], the comparison is between a consequence and the consequences of other actions in the same situation. In disappointment theory [4, 15], the comparison is between a consequence and the consequences of the same action in different situations. For example, in **Table 1**, if an agent selected action A_1 and situation S_1 occurred, it would feel disappointed, since situation S_2 has a better outcome. If the agent selected action A_2 and situation S_2 actually occurred, it would feel regretful, since having chosen action A_1 could have brought a better outcome. The agent could also experience both emotions when it chose action A_2 and the world turned out to be S_1.

Actions	S_1	S_2	S_3
A_1	\$100	\$200	\$0
A_2	\$50	\$100	\$50

Table 1. Outcomes of actions A_1 and A_2 for each possible situation

The above assumption does not explain the Ellsberg's paradox [9]: "*Two urns filled with red and black balls are before you. One has equal numbers of red and black balls, the other contains an unknown proportion. You need to choose an urn and a colour. If the ball is the colour you selected, you win a prize.*" People strongly preferred to choose the first urn, though an economic analysis suggests no reason to prefer one over the other, ie. the second urn is also considered to have equal proportion. This result suggests that the effect of psychological states such as regret and disappointment should not be studied by directly giving them a function and integrating this function with utility function. Instead, this depends upon the mode of psychological states when making a decision.

Modal logic has been an excellent analytical tool for the above problems. The semantics of the logic is a relational structure between *possible-worlds*. The truth value of any formula is evaluated inside the structure, at a particular (current) world. Modal operators provide access to information at other possible worlds, however, only worlds that are directly accessible from the current are allowed. Since the seminal work by Hintikka [11], mental attitudes such as knowledge, belief, desire, intention etc. have been formally modelled by modal logic. Though regret and disappointment theories have been intensively investigated with expected utility theory in economics, these works do suggest properties such as decision analysis for next immediate action, reference-point dependent [24] where a modal analysis should be taken into account more seriously.

3 EXPECTATION FORMALISM FOR DISAPPOINTMENT AND REGRET

3.1 Expectation logic

Expectation logic [26, 25] by Trần et al provides the first attempt to model human expectations using modal logic. Although the association between the two concepts expectation and observation remains as tight as we have in expected utility theory, their representations in expectation framework are different. In [26, 25], each observation carries some information about the environment that agent a_i is situated in. There are two closely related sources of the information: from the agent's set of sensors $\mathbb{S}_i$ and from the results of the agent's effectors $\mathbb{E}_i$. A possible world g is a way that the agent organises its sources to obtain certain information from the environment. Hence, each possible world represents a possible observation. Expectations are logical propositions describing the mental images of what can be obtained through an observation. The expectation language $\mathcal{L}$ is similar to the language of propositional logic augmented by the modal operator $\mathcal{E}_i$ and the observation operators $@_s$, where s is an observation label i.e. an atomic proposition which is $\mathtt{true}$ *at exactly one possible world in any model.*

Definition 1. (Expectation language) *Let Φ be a set of atomic expectation propositions. Let Ξ be a nonempty set of observation labels disjoint from Φ. An expectation language $\mathcal{L}$ over Φ and Ξ where $p \in \Phi$ and $s \in \Xi$ is defined as follows:*

$$\varphi ::= s \mid p \mid \neg\varphi \mid \varphi \wedge \varphi \mid \varphi \vee \varphi \mid \varphi \rightarrow \varphi \mid \langle \mathcal{E}_i \rangle \varphi \mid [\mathcal{E}_i]\varphi \mid @_s\varphi.$$

Let $\mathbb{G}_i$ be the set of observations for the agent a_i. Two observations are said to be related if one can be obtained by changing (adding/removing) the information sources (sensors/effectors) of the other. In other words, it is possible to reach the other observation by changing the information sources from the current one. Let $\sim_e^i \subseteq \mathbb{G}_i \times \mathbb{G}_i$ be the set of such related observations. The pair $\mathcal{F} = \langle \mathbb{G}_i, \sim_e^i \rangle$ is called an *observation frame*. The interpretation of an agent a_i's expectations is defined by the function $\pi : \Phi \cup \Xi \rightarrow \wp(\mathbb{G}_i)$. The crucial difference from orthodox modal logic in this definition is that for every observation label $s \in \Xi$, π returns a **singleton**. In other words, s is $\mathtt{true}$ at a unique observation, and therefore tags this observation [5]. The triple $\mathfrak{M} = \langle \mathbb{G}_i, \sim_e^i, \pi \rangle$ is called an expectation model.

Definition 2. *The semantics of expectation logic $\mathcal{L}$ are defined via the satisfaction relation $\models$ as follows*

1. $\langle \mathfrak{M}, g \rangle \models p$ *iff* $g \in \pi(p)$ *(for all* $p \in \Phi$*)*
2. $\langle \mathfrak{M}, g \rangle \models \neg\varphi$ *iff* $\langle \mathfrak{M}, g \rangle \not\models \varphi$
3. $\langle \mathfrak{M}, g \rangle \models \varphi \vee \psi$ *iff* $\langle \mathfrak{M}, g \rangle \models \varphi$ *or* $\langle \mathfrak{M}, g \rangle \models \psi$
4. $\langle \mathfrak{M}, g \rangle \models \varphi \wedge \psi$ *iff* $\langle \mathfrak{M}, g \rangle \models \varphi$ *and* $\langle \mathfrak{M}, g \rangle \models \psi$
5. $\langle \mathfrak{M}, g \rangle \models \varphi \rightarrow \psi$ *iff* $\langle \mathfrak{M}, g \rangle \not\models \varphi$ *or* $\langle \mathfrak{M}, g \rangle \models \psi$
6. $\langle \mathfrak{M}, g \rangle \models \langle \mathcal{E}_i \rangle \varphi$ *iff* $\langle \mathfrak{M}, g' \rangle \models \varphi$ *for some* g' *such that* $g \sim_e^i g'$
7. $\langle \mathfrak{M}, g \rangle \models [\mathcal{E}_i]\varphi$ *iff* $\langle \mathfrak{M}, g' \rangle \models \varphi$ *for all* g' *such that* $g \sim_e^i g'$
8. $\langle \mathfrak{M}, g \rangle \models s$ *iff* $\pi(s) = \{g\}$*, for all* $s \in \Xi$*,* g *is called the denotation of* s
9. $\langle \mathfrak{M}, g \rangle \models @_s\varphi$ *iff* $\langle \mathfrak{M}, g_s \rangle \models \varphi$ *where* g_s *is the denotation of* s*.*

where 1 – 7 are standard in modal logics with two additions of hybrid logics in 8 and 9.

Thus, if $[\mathcal{E}_i]\varphi$ is true in some state $g \in \mathbb{G}_i$, then if the agent a_i takes any further possible observation g' from g, φ will be its expectation at g'. For example, let φ be "the sun is shining". If Spirit is making observation g: taking pictures, then if it make a further observation g', either to use its camera again or roll ahead by its wheels, it will expect the sun is shining.

An observation statement $@_s\varphi$ says, whilst taking the observation named s, the agent a_i holds an expectation φ. The definition of the observation operator (item 9.) allows the agent to retrieve from another observation.

3.2 Formalising Regret and Disappointment

The theories of regret and disappointment by Bell [3, 4] and Loomes and Sugden [14] suggest that a difference while comparing decision outcomes and expectations triggers the two emotions. Regret originates from a comparison between the consequence of the selected observation and the consequences of other observations if they had been taken. Disappointment originates from a comparison between the prior expectation and the actual outcome of the same observation.

The first source of the above comparisons is the effects from effectors – *effective observations*. Since the agent simply releases the effects to the environment, it does not know immediately how the environment changes its effects. For example, Spirit rolls its wheels to intentionally move forward 0.5 metres. But the ground is slippery. It will not know how far it actually moves. The information about this result is purely non-intuitive and uncertain until it can be verified by the second source of comparison: sensory observations (primitive events obtained via sensors). Once Spirit takes another picture and compares this with previously obtained pictures it can figure out how far it has gone. We represent the organisation of such information sources by a sequence σ of sensors and effectors. Let $\varsigma \in \mathbb{S}_i$ be a sensor (e.g. Spirit's camera). Its first observation can be represented by $\sigma = \varsigma$. The next observation by subsequently rolling the wheels ϵ is represented by $\sigma' = \varsigma.(\varsigma)$. The brackets denote that the resulting effect ϵ is non-intuitive and this effect can be verified by Spirit's camera ς. We use $[\varsigma]$ to denote a generic observation (either sensory or effective observation). An assumption can be made by substituting a free variable x of the variable set $\mathbb{A}$ at some position of the sequence. Each of this sequence can be named using the observation labels Ξ by the naming function N.

Definition 3. *Let $\mathbb{A} = \{x, y, \ldots\}$ be a set of assumptions which are originally not bound to any sensors or effectors. Let Γ be a set of observation sequences. A linear observation method can be expressed by a string $\sigma \in \Gamma$ defined inductively as follows:*

i. *ς is an observation sequence for all $\varsigma \in \mathbb{S}_i$;*
ii. *If σ is an observation sequence, then so are $\sigma.\eta$ (if $\eta \in \mathbb{S}$) and $\sigma.(\eta)$ (if $\eta \in \mathbb{E}$);*
iii. *If σ is an observation sequence, then for all $x \in \mathbb{A}$ $\sigma.(x)$ is also an observation sequence but not $\sigma.x$. $\sigma.(x)$ stands for all possible successors of the observation sequence σ.*
iv. *$prefix(\sigma) = \{\tau \mid \sigma = \tau.\theta\}$ is a function which returns a set of all prefixes of an observation sequence σ.*
v. *The function $N : \Xi \rightarrow \Gamma$ assigns each label in Ξ to an observation sequence.*

By this addition, the following properties as essential for any observation frame. Firstly, once an agent a_i is created, its set of sensors and effectors should be considered as fixed (e.g. camera, wheels, arms…) However, the agent can extend its observation further by incorporating sensors and effectors from other agents in its observation (through mirror, tools…). But any extended observations should be ultimately rooted to the agent's innate set of sensors $\mathbb{S}_i$ (Definition 4(i)). Secondly, the interpretation should allow the justification of an expected sensing effect generated by an effector when it is possible to place the corresponding sensor for that effect (Definition 4(ii)). Thirdly, if any preceding observation sequence cannot be interpreted (explained), then there will be no interpretation for any subsequent observation based on the observation sequence (Definition 4(iii)). Fourthly, if the agent takes another sensory observation following an interpreted observation sequence then the new observation sequence must also be interpreted (Definition 4(iv)). Finally, by taking another observation based on the current observation sequence, the agent also associates its expectations with these observations (Definition 4(v)). Formally, these properties are stated as follows

Definition 4. *An observation interpretation is a pair $\langle \mathcal{F}, I \rangle$ where $\mathcal{F} = \langle \mathbb{G}_i, \sim_e^i \rangle$ is an observation method frame and I is a function $I : \Gamma \rightarrow \mathbb{G}_i \cup \{\bot\}$ which tells how the real world $\mathbb{G}$ is reflected into an agent's mind through the sequences of all available sensors and effectors in this observation frame. $\mathbb{G}_i$ is the reflected part of the global world $\mathbb{G}$ in the agent's a_i's mind through in this observation frame. An observation method interpretation function must satisfy the following properties:*

i. ***(Individuality)*** *$I(\eta) \in \mathbb{G}_i$ for all $\eta \in \mathbb{S}_i$;*
ii. ***(Justification)*** *$I(\sigma.(\eta)) = I(\sigma.\eta)$ for all $\sigma.(\eta)$ and $\sigma.\eta$ in Γ;*
iii. ***(Entirety)*** *for all $\sigma \in \Gamma$, if $I(\tau) = \bot$ for some $\tau \in prefix(\sigma)$ then $I(\sigma) = \bot$;*
iv. ***(Constructability)*** *for all labels $\tau.\eta \in prefix(\sigma) \cup \{\sigma\}$, if $I(\tau) \in \mathbb{G}_i$ then $I(\tau.\eta \in \mathbb{G}_i)$*
v. ***(Expectability)*** *$\tau = \sigma.[\eta], I(\sigma) \in \mathbb{G}_i, I(\tau) \in \mathbb{G}_i$ iff $I(\sigma) \sim_e^i I(\tau)$.*

For example, if the observation sequence $\sigma = \varsigma_1.\varsigma_2.\varsigma_3$ (where ς_1 is Spirit's side camera, ς_2 is its front camera, ς_3 is its microphone) is satisfied then $I(\varsigma_1), I(\varsigma_1.\varsigma_2)$, $I(\varsigma_1.\varsigma_2.\varsigma_3)$ must be defined. If we have an observation sequence $\sigma = \varsigma_1.(\varsigma_2).\varsigma_3$, where (ς_2) is the effect of rolling forward then $I(\varsigma_1.(\varsigma_2))$ need not be defined (i.e. $I(\varsigma_1.(\varsigma_2)) = \bot$) since that observation may not be captured or interpreted. However if it is, especially when σ exists, then by justification $I(\varsigma_1.(\varsigma_2).\varsigma_3)$ must be interpreted.

Without loss of generality, assume ψ is a formula representing a better outcome than $\neg\psi$. Regret and disappointment can be defined as follows

Definition 5. *Let $\varsigma_1, \varsigma_2 \in \mathbb{S}_i$ and $\varepsilon_1, \varepsilon_2 \in \mathbb{E}_i$ be the sensors and effectors of agent a_i, respectively. Also, let σ be any justified sequence of observations. Let s_1, s_2, t_1, t_2 be the observation labels for $I(\sigma.\varsigma_1), I(\sigma.\varsigma_2), I(\sigma.(\varepsilon_1), I(\sigma.(\varepsilon_2))$ respectively.*

i. *Regret: $(@_{s_2} t_1, @_{s_2} \neg\psi, @_{s_1} \psi)$;*
ii. *Disappointment: $(@_{s_2} t_2, @_{t_2} \psi, @_{s_2} \neg\psi)$.*

To illustrate this definition, let σ be a sequence of communication between Spirit Mars Rover and the land controller instructing it to explore its surroundings. Let $\varepsilon_1, \varepsilon_2$ be rolling and turning actions respectively. Let ψ represent "No obstacle ahead." Now if Spirit chooses to roll forward (t_1), but its front camera captures an obstacle in sensory observation s_2, whereas the side camera captures no obstacle in observation s_1. Spirit should experience regret in this case. Conversely, it would also feel disappointed when it chooses to turn around (t_2) with hope to see no obstacle but once the action is executed, its front camera still captures an obstacle.

4 REGRET/DISAPPOINTMENT AVERSION IN EXPECTATION REASONING

Research in economics and cognitive science has showed that a rational agent can employ different strategies in order to anticipate or avoid future regret and disappointment [32]. In this section, we describe different characterizations of an observation system to realise the properties.

According to findings by Zeelenberg et al [32], people experience regret usually feel they should have known better. Hence they tend to correct themselves by undoing the unpleasant effects. This means a symmetric observation frame is preferable.

$$s \rightarrow [\mathcal{E}_i]\langle \mathcal{E}_i \rangle s$$

In such frame, if an agent makes any observation and move to t it can back track to the previous observation s. A symmetric observation frame however is often too strong. Sometimes, it is not necessary or impossible to backtrack: the agent can use up its energy; its observation strongly related to time; there is only one single path, etc. In such cases, an Euclidean observation frame (for each observation) should be appropriate.

$$@_s\langle\mathcal{E}_i\rangle t \rightarrow @_s[\mathcal{E}_i]\langle\mathcal{E}_i\rangle t$$

In this type of observation frame, if an agent made a wrong decision that moves it from s to u. If it regrets since from s making an observation to t is the best way, it can make another observation to move from u to t.

The findings [32] also tell us that when people experience disappointment, they tend to get away from the situation by turning away from the event. A serial observation frame **D** is therefore applicable.

$$@_s\neg\langle\mathcal{E}_i\rangle t \rightarrow @_s\langle\mathcal{E}_i\rangle\neg t$$

Within this frame, if an agent feels disappointed while making an observation in s, if it cannot take another observation t, it is still able to take another further observation u, trying to get away from s. However, sometimes taking another observation does not mean you will not see the same situation as you have been seeing. To avoid this, (at least in short-term) the agent can choose an irreflexive observation frame.

$$s \rightarrow \neg\langle\mathcal{E}_i\rangle s$$

The above characterizations are mainly about the properties of observation frames. However, what we are also interested in is the construction of such frames. Analytic tableaux method [23] has been increasingly recognised as a useful tool in automated reasoning system. A tableau is a finitely branching tree whose nodes are labelled with tableau formulae. The reasoning process is simulated through tableau construction by applying inference rules. We now examine how human strategies when constructing their observation frame are implemented in observation calculus [26]. Human agents usually adopt two general strategies [32]:

1. ***People avoid making decisions***. The less decision one needs to make, the more one can prevent regret and disappointment. Each decision point is usually represented by a branching point in tableaux system. The only branching rule in **KE** system [7] adopted in Trần et al's expectation framework [26] is Principle of Bivalence (**PB**) rule. Hence, the number of branchings (space complexity) which require decision making is significantly reduced [7].
2. ***People also tend to delay their decision making***. Delay is usually used to gather more information relevant to the decision, with a view to make better decision. So when one has taken everything into account, it is less likely to experience regret. A technique by Fitting [10], which defers the choice of free-variables until more information is available, has been used to reduce search space and the non-determinism inherent in automated proof search. This technique resembles the ability to use expectations as assumptions to delay a current obstructed observation until justified. Among different approaches using free variables in the labels of semantic modal tableaux, Beckert and Goré's string matching technique [2] can be used to describe the connection between sensory observations and effective observations as above.

5 DISCUSSION

A common approach when using modal logic in formal analysis of mental attitudes is to give a modal operator for each attitude. Then, the relationship of these attitudes are studied through interaction axioms. Belief-Desire-Intention (BDI) by Rao and Georgeff [20] is one of the most well-known studies using this approach. Roughly speaking, beliefs represent the agent's current information about the world; desires represent the state of the world which the agent is trying to achieve; and intentions are chosen means to achieve the agent's desires. Following the philosopher Bratman [6], Rao and Georgeff formalised the constraints (Asymmetry thesis, non-transference and side-effect free principles) between these attitudes in [19]. The major drawback of this approach is to find a ground for such interactions. For example, it is difficult to give an analysis of why a very hungry eagle is still chasing its prey though it believes that the chase would take all of its remaining energy. It is even more difficult to explain when an information should be considered belief or knowledge. Expectation and observation logic [26, 25] establishes such grounds. As demonstrated in [25], Belief-Desire-Intention can be translated into the language of observation and expectation. In this work, we take a similar approach to describe the two other attitudes regret and disappointment without introducing new modal operators for the language.

In agent system, an agent often has no complete access to its environment. Hence, it is significant not only to represent such situations but also to describe the reasoning process an agent takes when dealing with them. $\mathcal{VSK}$ logic [31] is one of the efforts to formalise what information is true in the environment, what an agent can perceive and then know about its environment. A state of environment is captured into the agent's mind using *visibility function* (a.k.a *observation function* by van der Meyden [27]). Further, this work assumes "there is no uncertainty about performing an action in some state" and even claims that "Dropping this assumption is not problematic". However, we are unaware of any further work attempt to drop this assumption. In [26], actions are replaced by effective observations which can also be interpreted in many ways as sensory observations in [31] using an observation interpretation function. The justification property of this observation interpretation function however opens a new approach to drop the assumption. Furthermore, using the concepts of disappointment and regret, we suggested an approach towards constructing observation frames that can easily deal with unexpected situations.

6 CONCLUSION AND FURTHER WORK

Regret and disappointment are mental attitudes that strongly affect agent decision making. Though the idea has existed for many years [21, 22], until recently, the economists Bell [3, 4] and Loomes and Sugden [14] formalised them with the expected utility theory. However, their theories cannot predict situations such as Ellsberg's paradox [9]. In this paper, we presented a possibility of representing regret and disappointment using expectation framework by [26]. The work enables us to describe different characterisations of observation frames when an agent is discovering its environment. However, there are some open questions related to the construction of such observation frames during agent interaction with its environment.

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