Control of a Hydroforming Press with Bayesian Networks

Rainer Deventer, Heinrich Niemann [1] and **Martino Celeghini** [2]

Abstract. The demands to automatic control for industrial plants are growing due to an increased complexity of the manufacturing processes. To face these challenges, intelligent control is getting more and more important. For example, neural networks and fuzzy logic are regularly used. The usage of Bayesian networks is seldom mentioned even if many training algorithms are available and Bayesian networks are also able to act under real-time conditions. That means that main preconditions for a self-adaptive controller are given. This paper explains how a Bayesian network is employed as a controller. The main idea is to use the desired value as if it were already observed and to use marginalization for the calculation of the input. This principle is successfully applied to the control of a hydroforming press. As a result the process characteristics in terms of an uniform blank draw-in and the preforming pressure are improved.

1 INTRODUCTION

The current manufacturing technology has to face different challenges. The time to market is getting shorter and the production processes are getting more complex then ever. This development results in the need for self-adaptive and intelligent controllers. Many algorithms which are used mainly in the AI-community are applied in automatic control. Well-known examples are neural networks and fuzzy logic. Applications of Bayesian networks are seldom mentioned even if they offer attractive prerequisites and are used in many other domains, sometimes under real-time conditions [13, 12]. Examples are the application of Bayesian networks as spam-filter [11] and in medical expert systems [1]. In this paper the distribution of forces in a hydroforming press is modeled by a Bayesian network. The accuracy of the model is evaluated by cross-validation. As Bayesian networks define a unique distribution over random variables, there is no difference between input and output variables. Thus the desired values can be entered as evidence for the nodes representing the output. Afterwards the required input is calculated by marginalization. This approach only works with a perfect model. In reality there are a lot of external influences and it is tedious to include them all in the model. For example imagine that the room temperature of an office should be controlled. Beside the heating the temperature is influenced by the staff opening a window or a door. The external factors which affects the intended control are called *disturbance values*. To be prepared for changes in the environment it is necessary to model also the influence of the disturbance values [5].

Usually the disturbance value cannot be observed, but in the case discussed in this paper it is calculated as difference of the former output and the desired value. As a result of Bayesian control main process parameters of hydroforming are improved. Particularly the preforming pressure is increased and a uniform draw-in of the blank into the mould is achieved. Both characteristics improve the behavior in subsequent processing steps.

The paper will start with a short introduction to the application, so the reader, who is not familiar with hydroforming, will be able to understand the remaining parts. Afterwards the most important points about Bayesian networks are listed. The section about Bayesian networks is restricted to networks with continuous nodes, as the flow of forces can accurately be modeled by linear relationships. Bayesian networks that use solely continuous nodes are also called Gaussian networks. For more complex models hybrid Bayesian models [4, 14] are applicable. At the beginning of section 4 different Bayesian models are compared by cross-validation. The model which makes acceptable predictions for the input variables is used for control purposes. The results of the experiments, discussed in section 5.1, are evaluated in section 5.2 by comparison to the status quo. The article finishes with a summary and suggestions for future research.

2 HYDROFORMING

New forming technologies have been developed in the last decades in order to satisfy the increasing demand for lighter and more complex parts, especially in the automotive industry. The hydroforming of unwelded sheet metal pairs enables the production of complex hollow parts with high geometric accuracy and improved mechanical properties in a reduced number of process steps [10]. The results of the forming process are mainly influenced by the internal pressure P_{hf} between the blanks and by the blank holder force of the press required to provide tightness [9]. For complex geometries the distribution of the clamping force in the flange area is determinant for the hydroforming process [8]: high local pressure has to be applied to areas where a retain effect on the blank is needed and low contact pressure is desired where more draw-in of the blank into the die is required. For axially symmetric parts however the material flow is even more sensitive to the distribution of the contact pressure in the blank holder. In case of an asymmetrical distribution of the forces, an inhomogeneous draw-in of the blank occurs and premature failure by tearing is observed. Therefore a good forming result for symmetrical parts can be only achieved if the real contact pressure distribution is almost uniform. This can be obtained by controlling the distribution of the contact pressure in the blank holder. For this purpose many technical solutions have been developed allowing the process designer to influence the pressure distribution in the blank holder during the hydroforming process. In the modern hydraulic presses four up to six pistons are usually already placed inside the press ram and can be used to get the desired contact pressure distribution, if connected to the tool's blank holder by pillars and plates (e. g., for the tool in figure 1). Using this solution the pistons do not have to

[1] Friedrich–Alexander–Universität Erlangen–Nürnberg, 91058 Erlangen, Germany Chair for Pattern Recognition, email: {deventer,niemann}@informatik.uni-erlangen.de
[2] Chair of Manufacturing Technology, email: celeghini@lft.uni-erlangen.de

be integrated in the forming tool, which means a simpler design, but it is not simple to predict the effects of the ram pistons on the contact pressure distribution in the tool flange. This information can be supplied by a specific measurement set-up which is integrated in the tool [2]. Four load cells are positioned below the lower blank holder, equally spaced in the tool flange (figure 2) in order to determine the force flow in the forming tool. By this way the force flow of the considered press-tool system can be described by four input variables (forces of the ram pistons $F_{\mathrm{fl}}, F_{\mathrm{rl}}, F_{\mathrm{rr}}, F_{\mathrm{fr}}$) and four output variables (forces of the load cells $F_{\mathrm{l}}, F_{\mathrm{re}}, F_{\mathrm{ri}}, F_{\mathrm{f}}$). In order to model the force flow of the tool used in the forming trials two different training sets have been used. In the first training set equal forces at the cylinders are used. This training set is close to the usual process parameters. Due to the reduced drawing depth in the lower tool half, the contact between the sheet and the tool bottom occurs at very low pressure values. By this way the force resultant from the inner pressure on the lower sheets is directly transmitted to the tool cavity and does not influence the load cells under the blank holder. Consequently, by increasing the inner pressure P_{hf} the force resultant from the pressure on the upper sheet acts against the blank holder force and thus the measured force decreases.

For the second data set different forces are used at the cylinders. These process parameters are necessary to model the flow of the forces. But this time leakages occur at an early state of the process, thus the data are not suitable to model the influence of the pressure. This means that both data sets are necessary for training of the Bayesian model.

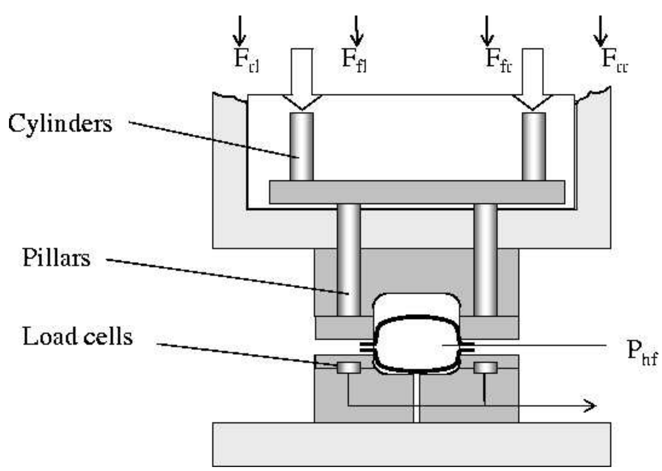

Figure 1. Hydroforming press (side view)

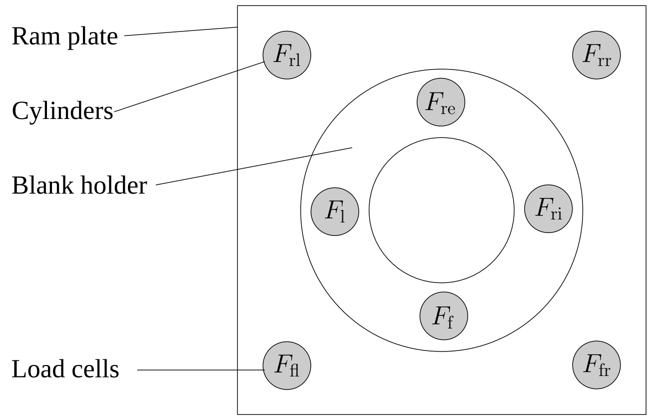

Figure 2. Measurement points (top view)

3 BAYESIAN NETWORKS

In the last section, the hydroforming press together with the measured signals are shortly introduced. The aim is to develop a stochastic model of the relationship between the cylinder forces $F_{\mathrm{rl}}, \cdots, F_{\mathrm{fl}}$ and the forces at the load cells $F_{\mathrm{re}}, \cdots, F_{\mathrm{l}}$. After training the model is used to calculate suitable inputs to obtain equal forces at all load cells. One possibility to model the press is a Bayesian model. A simple example is given in figure 3. Here each sensor output is modeled by a random variable represented by a node in the directed acyclic graph. A (conditional) probability distribution p, which depends also on the parent nodes, is associated to each node. E.g., the distribution $p(F_{\mathrm{f}}|F_{\mathrm{fl}}, F_{\mathrm{fr}}, P_{\mathrm{hf}})$ is associated to the first node in the second layer.

Generally the distribution in a Bayesian network is calculated by the *chain rule*

$$p(y_1, y_2, \cdots, y_n) = P(y_1) \cdot \prod_{i=2}^{n} p(y_i|\mathbb{p}(i)), \tag{1}$$

where y_i denotes the instantiation of the random variable Y_i and $p(y_i|\mathbb{p}(i))$ denotes the instantiations of the parents $\mathbb{p}(i)$ of the random variable Y_i. At the beginning of the development of Bayesian networks all random variables are assumed to be discrete [15], but currently also hybrid Bayesian networks [14], where discrete and continuous random variable are used at the same time, are employed. A normal distribution

$$p(y|\boldsymbol{z}) = \mathcal{N}(\alpha + \boldsymbol{\beta}^{\mathrm{T}}\boldsymbol{z}, \gamma) \tag{2}$$

which depends on three parameters $\alpha, \boldsymbol{\beta}$, and γ is associated to each continuous node. The mean of the normal distribution depends on a parameter α, and a weight vector $\boldsymbol{\beta}$ which is multiplied by the instantiation $\boldsymbol{z}$ of the parents of Y. The (co)variance is denoted by γ.

During the training process the parameters $\alpha, \boldsymbol{\beta}$, and γ are adapted by the EM algorithm [3] so that there is a maximal probability for the data given the parameters.

After the training an inference algorithm (usually the junction tree algorithm [14]) can be used to calculate the distribution $p(y|\boldsymbol{o})$ given the observations $\boldsymbol{o}$. This operation is called *introduction of evidence*. For example, the calculation of the input forces is done with the model depicted in figure 4. Here the desired outputs w and the disturbing value z are assumed to be known.

The second operation which is of importance is the *marginalization*. In many cases there are hidden nodes which cannot be observed or are not of any interest. For example, the second layer in figure 4 models the ideal output and is assumed to be unobservable. In this case the marginal distribution is calculated by integration

$$p(y_1, \cdots, y_{i-1}, y_{i+1}, \cdots, y_n) = \int_{-\infty}^{\infty} p(y_1, \cdots, y_n) dy_i \tag{3}$$

over all possible instantiations. If discrete random variables are used summation is used instead of integration. Using marginalization and the introduction of evidence the model depicted in figure 4 can be used to calculate the distribution of the forces at the cylinders given the desired values w and the disturbance z. Thus it is possible to calculate the input to obtain a desired output which is exactly the task of a controller. It is also possible to model dynamic systems with dynamic Bayesian networks. This is done with a similar approach, but this time the disturbance is estimated as difference between the predicted and observed model output [5].

4 MODELING AND CONTROL

4.1 Model selection

The first point in the development of a model based controller is usually the development of the structure. When modeling the hydroforming press the first consideration is that all cylinders are controlled individually. Thus the four input variables are modeled as independent. Figure 1 shows that each of the cylinders might have an influence on the forces of the load cells. First tests were made with a fully connected model. That is the dashed edges in figure 3 are included. The training of the model is done with the EM-algorithm [16]. The weights of the nodes representing the forces at the load-cells $F_{\mathrm{re}}, F_{\mathrm{ri}}, F_{\mathrm{f}}, F_{\mathrm{l}}$ are initialized with $[-0.05\ 0.25\ 0.25\ 0.25\ 0.25]$. That is, it is assumed that the pressure has a negative effect on the forces at the load cells and that all cylinder forces have an equal influence on the forces of the load cells. The other parameters are initialized randomly.

After training the weights of the models were analyzed. The adapted weights of all output nodes showed a similar pattern. The influence of the input nodes $F_{\mathrm{fl}}, \cdots, F_{\mathrm{rr}}$ to the two neighboring output nodes is large in comparison to the weights to the two nodes further away. Usually two weights are between 0.4 and 0.6, the other two weights are between 0.05 and 0. For example the weights from the nodes $P_{\mathrm{hf}}, F_{\mathrm{fl}}, F_{\mathrm{rl}}, F_{\mathrm{rr}}, F_{\mathrm{fr}}$ to F_{re} are $[-0.3488\ 0.0634\ 0.5348\ 0.4251\ -0.0056]$. The first weight in this vector represents the influence of the pressure, i.e. an increased pressure leads to decreased forces. The sum of all weights is close to 1, that is there is no additional source of forces. As two weights are much smaller than the other ones, it was decided to work with a partially connected model, the dashed edges in figure 3 are omitted.

The next question to be answered concerns the offset α of the output nodes. An argumentation from the physical point of view leads to the conclusion that the forces at the load cells are zero if there are no forces at the cylinders. In contrary to this consideration a possibility to train the offset is an additional degree of freedom and might lead to better training results. To test the accuracy of the model the data

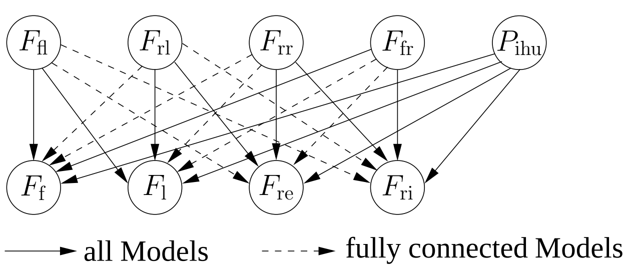

Figure 3. Models to be compared. Solid edges are element of all models, dashed edges are omitted in partially connected model.

set is split arbitrarily into a training set which contains 90% of the data and a disjoint validation set containing 10% of the data. After training of the model, the pressure P_{hf}, and the four cylinder forces $F_{\mathrm{fl}}, \cdots, F_{\mathrm{rr}}$ are entered as evidence and predictions are calculated for the four forces at the load cells $F_{\mathrm{f}}, \cdots, F_{\mathrm{ri}}$. For each variable the *relative error*

$$e_r = \frac{\mathrm{abs}(y_{\mathrm{p}} - y)}{y} 100\% \qquad (4)$$

is calculated. That is the relative error is proportional to the difference between the prediction y_p and the actual value y. Similarly the relative error for the prediction of the cylinder forces $F_{\mathrm{fl}}, \cdots, F_{\mathrm{rr}}$ (The forces at the load cells $F_{\mathrm{f}}, \cdots, F_{\mathrm{ri}}$ and the pressure P_{hf} are entered as evidence) and the pressure P_{hf} (all forces are entered as evidence) are calculated. This procedure is repeated 10 times with different training and validation sets. The relative errors together with the standard deviations are summarized in tables 1 and 2. The first number is the relative error, the second number states the standard deviation of the error.

Table 1. Relative error in percent for the prediction of input variables

	F_{fl}	F_{rl}	F_{rr}	F_{rr}
α tr.	9.32±16.99	10.34±16.08	11.1±20.91	14.93±21.02
α = 0	16.17±18.44	17.78±17.37	21.48±20.27	9.55±9.62

α tr. = Offset of output-nodes trained

Table 2. Relative error in percent for the prediction of output variables

	F_{f}	F_{l}	F_{re}	F_{ri}
α tr.	6.34 ± 7.39	11.17 ± 17.99	9.64 ±9.02	10.07 ± 26.89
α = 0	6.25 ±7.25	39.94 ± 51.69	35.61 ±111.98	34.70 ± 57.75

α tr. = Offset of output-nodes trained

A comparison between the model with a fixed offset α of the output-node with the model with a trained offset shows that it makes no sense to fix the mean of the output node. The simple explanation is that the mean of the output-nodes is an additional parameter to be adapted, so the regression with this additional parameter is more successful.

The prediction of the pressure results in a very large relative error (approximately 3300% with a standard deviation between 6900 and 8700%). The reason is that there are a lot of examples with a pressure close to zero (e.g. all the examples with different input forces). Thus even a small error might result in a great relative error. That is the relative error is no sensible mean to judge the accuracy of the prediction for the pressure P_{hf}.

The model shows that an increasing pressure leads to decreasing forces at the load cells. This effect can not be neglected. After training the weights between the pressure P_{hf} and the forces $F_{\mathrm{f}}, \cdots, F_{\mathrm{ri}}$ take on values between -0.25 and -0.45. As the pressure increases up to 200 bar, a high pressure leads to force differences at the load cells of up to 90 kN for each load cell. As the pressure depends on the volume of the hydroforming fluid pressed between the blanks it cannot be regarded as a parameter which can be controlled directly, but as a disturbance value. That is the cylinder force must be increased in order to compensate an increasing pressure. The disturbance is calculated as difference between the desired and the observed output.

Thus the high relative error for the prediction of the hydroforming pressure does not matter. It is part of the observed disturbance value.

The discussion leads to the conclusion that a partially connected model is used as base for the controller. The means of the output nodes are trained to increase the accuracy. As the model evaluation is based on a not observed validation set, the model is also able to make predictions for yet unobserved examples. So it can be used for control purposes.

The next section shows how the model is extended to use it as a controller for the hydroforming press introduced in section 2.

4.2 Control

In section 4.1 it is shown that a prediction of the forces at the cylinders given the desired forces at the load cells is possible. To model the hydroforming press, a partially connected net is applied. But a controller must not only be able to calculate suitable inputs under ideal conditions. It must react to disturbances or a changed environment. In control theory controllers are frequently triggered by errors, for example the deviation of the desired output from the required. The control model in figure 4 is therefore supplemented with additional nodes for the disturbance. According to control theory [17] it is possible (for linear systems only) to sum all disturbance variables z' to one variable z which is added to the output. For the hydroforming press this results in

$$w = F + z, \tag{5}$$

where z denotes the disturbance and w the observed output at arbitrary positions. This consideration results in the third and fourth layer of figure 4. According to equation (5) the edges $F \rightarrow w$ and $z \rightarrow w$

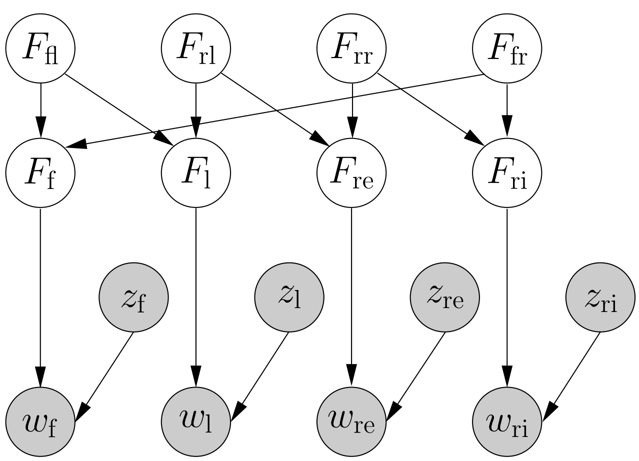

Figure 4. Bayesian controller for the hydroforming press

between the estimated output-force respectively the disturbance and the measured output force both have weight 1. The disturbance is calculated as difference between the measured force and the desired value. Thus both the desired values w and z are regarded as observed, they are marked by shaded nodes in figure 4. The node representing the pressure is no longer needed, as the influence of the pressure P_{hf} is included in the disturbance. An increased pressure leads to a negative disturbance. To keep the output close to the desired value the forces at the cylinders are increased.

The control cycle works as follows: After the observations are entered as evidence, the required input is calculated by marginalization and sent via an DA-converter to the cylinders of the hydroforming press. After the new inputs are calculated, the output is measured once again, so that the next control cycle takes place. In the next section 5.1 the experiments executed with the hydroforming press are explained and at the end in section 5.2 the results are discussed.

5 Experiments

5.1 Experimental setup

The forming trials were performed using an axially symmetric cup tool for double sheet hydroforming with inner diameter of 150 mm, with a drawing depth equal to 50 mm. The blank diameter was 250 mm and the material used was the mild steel FeP04 with a thickness of 1.5 mm. The total blank holder force was set to 700 kN, which is an optimal value for the considered part. The experiments were performed alternatively with and without blank holder force control, in order to clearly show the effects on the forming results. Furthermore the blanks were placed not in the middle of the tool, but displaced with respectively 5 mm and 10 mm offset from the tool center. This offset induces an asymmetrical distribution of the contact pressure in the blank holder, which results in an asymmetrical draw-in of the blank. The forming trials were stopped at the maximum pressure $P_{\mathrm{hf}}^{\max}$ achievable at the given blank holder force, when leakages of hydroforming fluid occur. In order to compare the forming results with and without control of the blank holder force, both tightness limit and blank draw-in were measured for the formed parts. The higher the tightness limit pressure and the more homogeneous the blank draw-in, the better is the forming of the part. The results are discussed in the next section.

5.2 Results

To test the control described in section 4.2 the blank on the top was inserted with a displacement of 5 and 10 mm. To evaluate the Bayesian controller the results are compared with the status quo at the chair of manufacturing technology, i.e. equal forces are used for all four cylinders. The results are listed in table 3. The first row shows that the maximal forces $\Delta F_{\max}$ are halved. The same is valid when the mean values of the maximal forces during the complete production $\overline{\Delta F_{\max}}$ are compared. Also in that case the maximal difference between the forces is halved, both for the displacement of 5 and 10 mm. The equal distribution of the forces has a positive influence on

Table 3. Results of control

	5 mm displacement		10 mm displacement	
	NC	BNC	NC	BNC
$\Delta F_{\max}$ [kN]	41.5	26.0	45.8	26.4
$\overline{\Delta F_{\max}}$ [kN]	32.9	18.0	38.2	21.1
$\overline{P_{\mathrm{hf}}^{\max}}$ [bar]	180.4	205.1	154.0	164.4
$\lvert s_{0^\circ} - s_{180^\circ}\rvert$ [mm]	2.9	2.7	6.0	1.4
$\lvert s_{90^\circ} - s_{270^\circ}\rvert$[mm]	12.4	5.2	24.9	15.5

BNC Control with Bayesian network
NC Equal forces at cylinder

the flow of the blank into the form. The forth and fifth row of table 3 lists the difference of the blank draw-in s between the front s_{0° and the rear s_{180° (row 4) and the left s_{90° and the right s_{270°. The displacement of the blank is included in the fifth row. That is two thirds of the difference of 15.5 mm mentioned in the last column dates back to the displacement of 10 mm. It gets clear that the blank is drawn more uniformly into the form if the hydroforming press is controlled by a Bayesian network.

An additional advantage of the control is the higher preforming pressure listed in the third row of table 3. Usually a higher preforming pressure means that more blank is drawn into the form. That is the minimal thickness of the blank is higher, so that a higher pressure during calibration can be used.

That is all three quality criteria are improved by the usage of a Bayesian network. As the controller is based on the very general idea to enter both the desired value and the disturbance as evidence and to calculated the input by marginalization it is expected that this idea is transferable to other stochastic models and control tasks.

6 SUMMARY

The main idea of a Bayesian controller is to model the dependency between the input and output variables. After the model is readily trained the desired value is entered as evidence and the input variables are gained by marginalization. This principle is successfully applied to the model of a hydroforming press to control the distribution of the forces at the blank holder.

The model consists of the structure, i.e. the dependency of the random variables. There are many possible sources for the structure. When manufacturing processes are modeled, the knowledge of the engineers can be used to get an initial idea of the structure. The parameters can be trained by the EM-algorithm. For the training of the model it is necessary to use different input signals, so that the relationship between multiple inputs and outputs is learned.

Different structures are judged by the relative error of the prediction. For the calculation of the relative error cross validation is used, so that the capability to make predictions for yet unknown examples is tested. Of course the discussion with engineers is only one possibility, there is a rich source of literature about structure learning, e.g. [6, 7]. But the reader should keep in mind, that the distribution of the input variables is not arbitrary, but stems from a test plan. Thus a dependency between input variables should be avoided.

As a result of the Bayesian controller the force difference at the blank holder is halved, the draw-in of the blank is more uniform and the preforming pressure is increased. Thus the usage of a Bayesian controller is a great improvement in comparison to the status quo.

But the current blank geometry used for the experiments is relatively simple. Further experiments are needed with more complex forms.

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