# A Syntactical Approach to Revision

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**Abstract.** The aim of this article is to revisit Dalal's operator for belief revision. Dalal has proposed a technique for revising belief bases based on the minimization of a distance between interpretations. The result is a concrete operator that can be considered either from a semantical point of view (distance between interpretations) or from a syntactical point of view (number of atoms that have their truth values changed). Dalal has shown that the so-called Alchourrón, Gärdenfors and Makinson (AGM) postulates are satisfied by its operator. The AGM postulates constrain the revision process so that minimal changes occur in the belief set. In this article, our contribution is twofold: first, we improve Dalal's algorithm by avoiding multiple satisfiability checking, which are NP-complete tasks. Our algorithm requires only one NP-stage if beliefs are expressed in a specific syntax, namely the prime implicates and prime implicants. Second, we propose a new distance based on the number of prime implicates in contradiction with the incoming new information. We argue that in some cases changing a minimal set of propositional symbols do not necessarily entail minimal changes.

## 1 INTRODUCTION

The description of the dynamics of beliefs are mainly influenced by the so-called *AGM postulates* [1]. These postulates states that minimal changes should occur in the initial belief base in order to introduce in a consistent way a conflicting statement. Following these principles, several operators have been proposed [2, 11, 12, 13, 16]. In [6], Dalal proposes a theory of knowledge revision based on the AGM principles. Dalal defines a semantic measure for minimal change and introduces a syntactical revision operator, that respects the semantical definition. The intuitive idea behind Dalal's notion of minimal change is to change the smallest number of propositional symbols truth values. This notion is also shared in several contributions both in the belief revision area and in the belief update area (see [14] for a review). The revision technique proposed by Dalal has some caveats. First, it requires multiple satisfiability checking which is a NP-complete problem. Second, changing one propositional symbol truth value may lead to significant changes if this symbol frequently appears in the formulas of the initial belief base. Thus, the notion of minimal change which seems to be fair according to Dalal is actually biased by the structure of the belief base.

In this paper we improve Dalal's revision operator by avoiding these two problems. First, we avoid these multiple satisfiability checking by expressing beliefs in a specific syntax. We show that if beliefs are represented with sets of prime implicates and prime implicants, Dalal's revision operator can be deeply improve since it requires only one NP task. Second, we propose a new notion of minimal change based on the number of prime implicates concerned by a propositional symbol.

The paper is organized as follows: in section 2, we present logical definitions of prime implicates and prime implicants in terms of disjunctive normal forms and conjunctive normal forms. In section 3, we present an algorithm for computing prime implicates and prime implicants of a belief base when this belief base is expressed as a set of clauses. In section 4, we recall the AGM postulates and the Dalal's procedure for belief revision. In section 5, we revisit Dalal's operator by, first, improving the algorithm and, second, improving the notion of minimal change in Dalal's framework. Section 6 concludes the paper by considering some open issues.

## 2 PRELIMINARIES

Let $P = \{p_1, \ldots, p_n\}$ be a set of propositional symbols and $LIT = \{L_1, \ldots, L_{2n}\}$ the set of their associated literals, where $L_i = p_j$ or $L_i = \neg p_j$. A *clause* $C$ is a *disjunction* [9] of literals: $C = L_1 \vee \cdots \vee L_{k_C}$ and a *dual clause*, or *term*, is a *conjunction* of literals: $D = L_1 \wedge \cdots \wedge L_{k_D}$.

Given a propositional logic language $\mathcal{L}(P)$ and an *ordinary formula* $\phi \in \mathcal{L}(P)$, there are algorithms for converting it into a *conjunctive normal form (CNF)* and into a *disjunctive normal form (DNF)* (e.g., [18, 20, 21]). The CNF is defined as a conjunction of clauses, $CNF_\phi = C_1 \wedge \cdots \wedge C_m$, and the DNF as a disjunction of terms, $DNF_\phi = D_1 \vee \cdots \vee D_w$, such that $\phi \equiv CNF_\phi \equiv DNF_\phi$.

A clause $C$ is an *implicate* [14, 15, 17] of a formula $\phi$ iff $\phi \models C$, and it is a *prime implicate* iff for all implicates $C'$ of $\phi$ such that $C' \models C$, we have $C \models C'$, or syntactically [19], for all literals $L \in C$, $\phi \not\models (C - \{L\})$. We define $PI_\phi$ as a conjunction of prime implicates of $\phi$ such that $\phi \equiv PI_\phi$. A term $D$ is an *implicant* of a formula $\phi$ iff $D \models \phi$, and it is a *prime implicant* iff for all implicants $D'$ of $\phi$ such that $D \models D'$, we have $D' \models D$, or syntactically, for all literals $L \in D$, $(D - \{L\}) \not\models \phi$. We define $IP_\phi$ as a disjunction of prime implicants of $\phi$ such that $\phi \equiv IP_\phi$. In propositional logic, implicates and implicants are dual notions, in particular, an algorithm that calculates one of them can also be used to calculate the other [5, 21].

Alternatively, prime implicates and implicants can be defined as special cases of CNF (or DNF) formulas, that consist of the smallest sets of clauses (or terms) closed for inference, without any subsumed clauses (or terms), and not containing a literal and its negation. In the sequel, conjunctions and disjunctions of literals, clauses or terms are treated as sets.

The prime canonical forms are important in knowledge representation, because theories compiled into them can be queried in polynomial time for consistency, validity, clause entailment, implicants, equivalence, sentential entailment and model enumeration [7].

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Given a formula $\phi$, represented by a conjunctive normal form $CNF_\phi$ and by a disjunctive normal form $DNF_\phi$, we introduce the concept of a *conjunctive quantum*, defined as a pair $(L, F_c)$, where $L$ is a literal that occurs in $\phi$ and $F_c \subseteq CNF_\phi$ is its set of *conjunctive coordinates* that contains the subset of clauses in $CNF_\phi$ to which literal $L$ belongs. Dually, we define a *disjunctive quantum* as a pair $(L, F_d)$, where $L$ is a literal that occurs in $\phi$ and $F_d \subseteq DNF_\phi$ is its set of *disjunctive coordinates* that contains the subset of terms in $DNF_\phi$ to which literal $L$ belongs. A quantum is noted $L^F$. The rationale behind the choice of the name *quantum* is to emphasize that we are not interested in an isolated literal, but that our *minimal* unit of interest is the literal and its situation with respect to the theory in which it occurs.

**Example 1** *Consider the theory $\psi$ given by the following CNF:*

$$\begin{array}{ll} 0 : \neg p_1 \vee p_2 \vee p_3 & 3 : \neg p_1 \vee \neg p_2 \\ 1 : p_4 \vee \neg p_2 \vee \neg p_3 & 4 : \neg p_1 \vee \neg p_3 \\ 2 : p_4 \vee \neg p_2 \vee p_3 & 5 : \neg p_3 \vee \neg p_2 \end{array}$$

*The literals that occur in $\psi$ can be represented by the following set of conjunctive quanta:* [3]

$$\{\neg p_1^{\{0,3,4\}}, p_2^{\{0\}}, \neg p_2^{\{1,2,3,5\}}, p_3^{\{0,2\}}, \neg p_3^{\{1,4,5\}}, p_4^{\{1,2\}}\}$$

The quantum notation can be used to characterize $PI_\phi$ and $IP_\phi$ of a formula $\phi$, given by one $CNF_\phi$ and one $DNF_\phi$. Let $D = L_1 \wedge \cdots \wedge L_k$ be a term represented by a set of conjunctive quanta, $L_1^{F_c^1} \wedge \cdots \wedge L_k^{F_c^k}$. $D$ is an implicant of $\phi$ if $\cup_{i=1}^k F_c^i = CNF_\phi$, i.e., $D$ contains at least one literal that belongs to each clause in $CNF_\phi$, spanning a path through $CNF_\phi$, and no pair of contradictory literals. To be a prime implicant, a term $D$ has to satisfy a *non redundancy* condition, i.e., each of its literals should represent *alone* at least one clause in $CNF_\phi$. To define this condition, we introduce the notion of *exclusive coordinates*. Given a term $D$ and a literal $L_i \in D$, the exclusive conjunctive coordinates of $L$ in $D$, defined by $\widehat{F}_c^i = F_c^i - \cup_{j=1, j\neq i}^k F_c^j$, are the clauses in set $F_c^i$, to which no other literal of $D$ belongs. Using this notion, the non redundancy condition can be written as: $\forall i \in \{1, \ldots, k\}, \widehat{F}_c^i \neq \emptyset$. The exclusive coordinates play an important role in the proposed revision method (see Section 5.2). Dually, a clause $C = L_1 \vee \cdots \vee L_k$ represented by a set of disjunctive quanta, $L_1^{F_d^1} \vee \cdots \vee L_k^{F_d^k}$, such that $\cup_{i=1}^k F_d^i = DNF_\phi$, with no pair of tautological literals allowed, is an implicate. Again $C$ is a prime implicate if it satisfies the non redundancy condition, expressed by $\forall i \in \{1, \ldots, k\}, \widehat{F}_d^i = F_d^i - \cup_{j=1, j\neq i}^k F_d^j \neq \emptyset$, where $\widehat{F}_d^i$ is the set of exclusive disjunctive coordinates of $L_i$ in $C$.

**Example 2** *Consider the theory $\psi$ introduced in example 1. The set $D = \{\neg p_1^{\{0,3,4\}}, \neg p_2^{\{1,2,3,5\}}, \neg p_3^{\{1,4,5\}}\}$ is an implicant of $\psi$ because the union of the conjunctive coordinates associated with its quanta is equal to the set of clauses in $CNF_\psi$. The exclusive conjunctive coordinates of the quanta in $D$ are given by: $\neg p_1^{\{0\}}, \neg p_2^{\{2\}}, \neg p_3^{\{\}}$. The fact that $\neg p_3$ has empty exclusive coordinates indicate that $D$ is not a prime implicant.*

Given a theory $\phi$, it is possible to determine the sets of conjunctive and disjunctive quanta that, respectively, define $IP_\phi$ with respect to $PI_\phi$ and $PI_\phi$ with respect to $IP_\phi$. This minimal quantum notation is an enriched representation for prime implicates and implicants sets, in the sense that it explicitly contains the "holographic" relation between literals in one form and the clauses (or terms) in which they occur in the other form.

[3] To simplify the notation, the sets of conjunctive coordinates contain the clause numbers instead of the clauses themselves.

## 3 PRIME IMPLICANTS/IMPLICATES

The proposed revision method makes intensive use of the prime implicants and the prime implicates of a formula represented in the quantum notation. In this section, we sketch an algorithm [3] that builds such representations. The basic idea of the algorithm is, given a propositional theory $\phi$ represented by $CNF_\phi$ (respectively, by $DNF_\phi$), calculate the set $IP_\phi$ (respectively $PI_\phi$). We describe the algorithm that generates $IP_\phi$ given $CNF_\phi$, the algorithm that generates $PI_\phi$ given $DNF_\phi$ being its exact dual.

The algorithm receives a CNF representation, calculates the conjunctive coordinates of all its literals, and begins a search in a state space where each state is represented by a set of quanta that represent an incomplete prime implicant: $D = L_1^{F_c^1} \wedge \cdots \wedge L_k^{F_c^k}$. Successor states are generated by adding to the set a new quantum, consistent with the quanta already present in the state and that respects the non redundancy condition (see Section 2). Each incomplete prime implicant $D$ has an associated *gap*, defined as the set of clauses to which none of its associated literals belong: $G_D = CNF_\phi - \cup_{i=1}^k F_c^i$.

The initial states are singletons, each one of them contains the quantum associated with a literal that belongs to one specific clause $C_i \in CNF_\phi$, e.g., a clause that contains the most frequent literal in $CNF_\phi$. Once an initial clause is adopted, the problem reduces to a set of independent search problems, one for each initial state, because any path through $CNF_\phi$ must pass exactly by one literal in clause $C_i$.

Finally, the final states are defined as those that correspond to *complete* prime implicants, i.e., those that span a complete path through $CNF_\phi$. This condition can be directly verified by the following property of the conjunctive coordinates of the associated quanta: $\cup_{i=1}^k F_c^i = CNF_\phi$ or $G_D = \emptyset$.

At each search step, usually several quanta would qualify as possible extensions to a given incomplete prime implicant. To avoid duplicate states, we restrict which quanta can be added using the following procedure. Let $D$ be an incomplete prime implicant and $S_D$ a set of quanta, each one of which can be used to extended $D$. Initially, we sort $S_D$ according to some *quality criterion*, e.g., maximal intersection of conjunctive coordinates with the state gap. Let $L_i^{F_i}$ and $L_j^{F_j}$ be two quanta in $S_D$, such that $L_i^{F_i}$ is better than $L_j^{F_j}$. The new state obtained by adding $L_i^{F_i}$ to $D$ is allowed to be extended in the future with $L_j^{F_j}$, but the state obtained by adding $L_j^{F_j}$ to $D$ is not allowed to be extended by $L_i^{F_i}$. This means that each state $D$ must remember its origins, in the form of a list $X_D$ of *forbidden* quanta[4].

**Example 3** *Consider a state $D$ with a list of forbidden quanta $X_D$ and a set of possible extensions given by $S_D = \{L_1^{F_1}, L_2^{F_2}, L_3^{F_3}\}$, where $S_D$ is already sorted according to the adopted quality criterion. The possible successors states are:*

- $D_1 = D \cup \{L_1^{F_1}\}$ *with* $X_{D_1} = X_D \cup \{\overline{L_1}^{\overline{F_1}}\}$
- $D_2 = D \cup \{L_2^{F_2}\}$ *with* $X_{D_2} = X_D \cup \{\overline{L_2}^{\overline{F_2}}, L_1^{F_1}\}$
- $D_3 = D \cup \{L_3^{F_3}\}$ *with* $X_{D_3} = X_D \cup \{\overline{L_3}^{\overline{F_3}}, L_1^{F_1}, L_2^{F_2}\}$

*where $\overline{L_i}^{\overline{F_i}}$ is the quantum associated with literal $\neg L_i$.*

Besides not including contradictory nor redundant literals, each state should not be extended by a quantum that generates a contradic-

[4] The non contradiction condition test can also be implemented using the same list of forbidden quanta, it is only necessary to add to this list the negation of each quantum included in the state.

tion with respect to the gap clauses, i.e., for each state $D$, the following theory in CNF must be consistent: $\{C - \overline{D} \mid C \in G_D\}$, where, given a clause or a term $A$, we note $\overline{A}$, the clause or term obtained from $A$ by flipping all its literals. This condition can be extended to take into account, not only the negation of the literals in the state, but also the literals in the forbidden list $X_D$ (that already includes $\overline{D}$). The new theory that must be consistent is: $\{C - X_D \mid C \in G_D\}$.

This additional restriction greatly reduces the number of successor states, because the forbidden list includes not only the negation of state literals, but also all literals that are not included in the state to avoid state repetition and those that were detected as potentially contradictory when the algorithm tried to extend the state with them. This non contradictory theory, analogously to the Davis-Putnam algorithm [8], is simplified at each step by unit resolution and subsumption and all the non redundant literals that occur in it as unitary clauses are included simultaneously into the state, further reducing the number of successors.

**Example 4** *Consider the theory $\psi$ introduced in example 1. Using the proposed dual transformation algorithm, it is possible to determine the following set of prime implicants, represented as sets of quanta:*

$$\begin{array}{l} 0 : \neg p_1^{\{0,3,4\}} \wedge \neg p_3^{\{1,4,5\}} \wedge p_4^{\{1,2\}} \\ 1 : \neg p_1^{\{0,3,4\}} \wedge \neg p_2^{\{1,2,3,5\}} \end{array}$$

*One more application of the dual transformation*[5] *determines the prime implicates. The pair $(PI, IP)$ corresponding to the theory, in quantum notation is given by:*

| $PI$ | $IP$ |
|---|---|
| $0 : \neg p_2^{\{1\}} \vee \neg p_3^{\{0\}}$ | $0 : \neg p_1^{\{2\}} \wedge \neg p_3^{\{0\}} \wedge p_4^{\{1\}}$ |
| $1 : \neg p_2^{\{1\}} \vee p_4^{\{0\}}$ | $1 : \neg p_1^{\{2\}} \wedge \neg p_2^{\{0,1\}}$ |
| $2 : \neg p_1^{\{0,1\}}$ | |

# 4 REVISION

To change an agent's belief base, we can either add new belief or delete a previously existing belief [12]. The first characteristic of belief revision is that we need extra-logical criteria in order to decide which sentences should be retracted or kept among the multiples choices. The second characteristic concerns the change function: general properties may be asserted even if we do not define the function. Such properties are described by the AGM postulates [1]: they describe some prerequisites for the belief contraction and revision functions. These postulates describe how changes should occur based on the following main principles: minimal change and syntax independence.

## 4.1 AGM Postulates

Given a belief base represented by a theory $\psi$, an interpretation $w$ is a truth assignment to all the propositional symbols that occur in $\psi$. If $\psi$ is true in $w$, then $w$ is a model of $\psi$, i.e., $w \in mod(\psi)$ where $mod(\psi)$ is the set of all models of $\psi$. Given a new information $\mu$ that contradicts $\psi$, the revised belief base $\psi \circ \mu$ is obtained by minimally changing the models of $\psi$ in such a way that $\mu$ holds in at least some of them. According to [16], a revision function should satisfy the following postulates:

[5] In fact, this second application is not necessary, because, once the prime implicants are known, there are polynomial time algorithms to calculate the prime implicates [7].

**(R1)** $\psi \circ \mu$ implies $\mu$.
**(R2)** If $\psi \wedge \mu$ is satisfiable then $\psi \circ \mu \equiv \psi \wedge \mu$.
**(R3)** If $\mu$ is satisfiable then $\psi \circ \mu$ is also satisfiable.
**(R4)** If $\psi_1 \equiv \psi_2$ and $\mu_1 \equiv \mu_2$ then $\psi_1 \circ \mu_1 \equiv \psi_2 \circ \mu_2$.
**(R5)** $(\psi \circ \mu) \wedge \phi$ implies $\psi \circ (\mu \wedge \phi)$.
**(R6)** If $(\psi \circ \mu) \wedge \phi$ is satisfiable then $\psi \circ (\mu \wedge \phi)$ implies $(\psi \circ \mu) \wedge \phi$.

## 4.2 Dalal's Approach

In [6], Dalal defines *minimal change*, as the change in the truth value of only one propositional symbol, but not to be biased in favor of any one of them, he adopts as the smallest unit of change all changes in truth values of all possible single propositional symbols. This notion is formalized by the following function, where $A$ is a set of interpretations and $Dist(w, w')$ is the number of propositional symbols that take different truth values in $w$ and $w'$:

$$g^i(A) = \bigcup_{w \in A} \{w' \mid Dist(w, w') \leq i\}$$

In the same way, given the theory $\psi$, we can define the formula $G^i(\psi)$, through its models, by $mod(G^i(\psi)) = g^i(mod(\psi))$. $G^i$ can be seen as a generalization operator that takes a formula and returns a subset of its logical closure. Using these notions, Dalal defines the revision operator $\circ$ in such a way that $\psi \circ \mu = G^k(\psi) \cup \{\mu\}$, where $k$ is the least value of $i$ for which $\mu$ evaluates to true in some interpretation in the set $g^i(mod(\psi))$.

Dalal presents a technique to obtain $G^k(\psi)$ as a syntactical transformation of $\psi$. For each propositional symbol $p_i$, he defines the sets $\psi^+_{p_i}$ and $\psi^-_{p_i}$ such that (i) they do not contain $p_i$, and (ii) $\psi \equiv (p_i \wedge \psi^+_{p_i}) \vee (\neg p_i \wedge \psi^-_{p_i})$. These sets can be obtained by replacing $p_i$ by true or false, respectively, in $\psi$. He also defines the *resolvent* of $\psi$ with respect to $p_i$ as $res_{p_i}(\psi) = \psi^+_{p_i} \vee \psi^-_{p_i}$ and, finally, proves the following theorem:

$$G^i(\psi) = \begin{cases} \psi & \text{for } i = 0, \\ res_{p_1}(G^{i-1}(\psi)) \vee \ldots \vee res_{p_n}(G^{i-1}(\psi)) & \text{for } i > 0 \end{cases}$$

**Example 5** *Consider the theory $\psi$ of example 1 and the new information $\mu$, given by $PI_\mu = (p_1 \vee p_2) \wedge p_3$ and $IP_\mu = (p_1 \wedge p_3) \vee (p_2 \wedge p_3)$. Using $IP_\psi$, we calculate the resolvents for the propositional symbols that occur in $\mu$:*

$$res_{p_1}(\psi) = (\neg p_3 \wedge p_4) \vee \neg p_2$$

$$res_{p_2}(\psi) = (\neg p_1 \wedge \neg p_3 \wedge p_4) \vee \neg p_1$$

$$res_{p_3}(\psi) = (\neg p_1 \wedge p_4) \vee (\neg p_1 \wedge \neg p_2)$$

*Therefore, we get:*

$$G^1(\psi) = \neg p_1 \vee \neg p_2 \vee (\neg p_3 \wedge p_4)$$

*and the revised theory is given by:*

$$G^1(\psi) \cup \{\mu\} = (p_1 \wedge \neg p_2 \wedge p_3) \vee (\neg p_1 \wedge p_2 \wedge p_3)$$

# 5 PROPOSED APPROACH

The revision technique proposed by Dalal requires, at each step, a logical consistency verification between $G^i(\psi)$ and $\mu$. This verification, a satisfiability test, is a NP-Complete task and is one of the main drawback of Dalal's approach. We propose to avoid this multiple satisfiability checking, by representing the formula $\psi$ by prime implicants/implicates.

## 5.1 Prime Implicants

Let $\psi$ be a belief base and $\mu$ a new belief that is contradictory with $\psi$. First, we calculate the prime implicants of $\psi$ and $\mu$, given by $IP_\psi$ and $IP_\mu$, respectively, using the dual transformation algorithm.

*Only this first step is NP-Complete.*

Second, we calculate the following set of terms:

$$\Gamma = \{D \mid D = D_\mu \cup (D_\psi - \overline{D_\mu}), D_\mu \in IP_\mu \text{ and } D_\psi \in IP_\psi\}$$

It is possible to calculate, for each term $D$ in the set $\Gamma$, the number of literals that have been deleted from the associated $D_\psi$ in order to make it consistent with $D_\mu$. This number is given by $k_D = \mid D_\psi \cap \overline{D_\mu} \mid$. We define the revised belief base as the following DNF:

$$DNF_{\psi \circ \mu} = \{D \in \Gamma \text{ such that } k_D \text{ is minimal}\}$$

*This second step is polynomial time on the size of $IP_\psi$ and $IP_\mu$.*

The following theorem establishes that the proposed revised belief base is equivalent to the one defined in [6].

**Theorem 1** *Given a propositional belief base $\psi$ and a new contradictory information $\mu$, $G^k(\psi) \cup \{\mu\} \equiv DNF_{\psi \circ \mu}$, with $k = k_D$.*
**Proof:** *We assume, without lose of generality, that $\psi$ is represented by $IP_\psi$, then the definition of $G^k(\psi)$ can be written as:*

$$G^k(\psi) = \bigvee_{[p_k]} res_{[p_k]}(\psi)$$

*where the $[p_k]$'s are all the the subsets of $P = \{p_1, \ldots, p_n\}$ of size $k$ built up from the propositional symbols that occur in $\psi$. The definition of $res$ now becomes: $res_{[p_k]}(\psi) = \psi^+_{[p_k]} \vee \psi^-_{[p_k]}$, where $\psi^+_{[p_k]}$ and $\psi^-_{[p_k]}$ do not contain the propositional symbols in $[p_k]$ and are defined in such a way that:*

$$\psi \equiv (\bigwedge_{p \in [p_k]} p \wedge \psi^+_{[p_k]}) \vee (\bigwedge_{p \in [p_k]} \neg p \wedge \psi^-_{[p_k]})$$

*Given the definition of $k_D$, for $k < k_D$ each $res_{[p_k]}$ contains at least one literal that is contradictory with $\mu$ and therefore $G^k(\psi) \cup \{\mu\}$ is contradictory. On the other hand, for $k = k_D$, the set of those $res_{[p_k]}$ that are not contradictory with $\mu$ correspond exactly to the elements of $DNF_{\psi \circ \mu}$.* $\Box$

**Corollary 1** *The syntactical definition of the revision operator $\circ$ satisfies* **R1∼R6***, as defined in Section 4.1.*

## 5.2 Another Minimum

Dalal's definition of minimal change considers the truth value of a propositional symbol as the minimal information chunk and explicitly decides not to be biased in favor of any one of them. We claim that this is a sensible choice only if the belief base $\psi$ consists of a conjunction of literals. If $\psi$ is a more complex formula, e.g., a conjunction of clauses, then the relative importance of a literal in a given model of $\psi$ is already biased and depends on the structure of the formula, in the sense that flipping one or another propositional symbol truth value may cause quite different effects on the formula.

In Section 5.1, we choose the terms of the revised belief base $DNF_{\psi \circ \mu}$ among those terms $D \in \Gamma$ that have the minimum $k_D$ and proved that this choice corresponds to Dalal's notion of minimal change. To take into account the structure of $\psi$, we assume that $\psi$ is represented by $PI_\psi$ and $IP_\psi$ and observe that, according to the quantum notation, each literal in a term $D_\psi \in IP_\psi$ represents a certain number of clauses in $PI_\psi$. Next, we assume that a clause in the conjunctive set $PI_\psi$, which is unique and non subsumed by any other, corresponds better to the idea of a *knowledge unit* than a literal in $D_\psi$. Finally, we choose to include in the revised base the terms $D$ that are associated with sets $D_\psi \cap \overline{D_\mu}$ whose literals have the smaller set of exclusive conjunctive coordinates. This can be formalized as follows. Let $D_\psi \cap \overline{D_\mu} = \{L_1^{F_c^1}, \ldots, L_k^{F_c^k}\}$ be the set of literals of $D_\psi$ that conflict with $D_\mu$ represented in quantum notation and $\widehat{F}_c^i$ their associated exclusive coordinates (see Section 2). We define $\widehat{k}_D = \mid \cup_{i=1}^k \widehat{F}_c^i \mid$ and introduce a new revision operator $\widehat{\circ}$ defined by:

$$DNF_{\psi \widehat{\circ} \mu} = \{D \in \Gamma \text{ such that } \widehat{k}_D \text{ is minimal}\} \qquad (1)$$

This new operator measures the degree of change, with respect to a model, by the number of clauses in $PI_\mu$ that are invalidated by flipping a propositional symbol truth value.

**Example 6** *Consider the same theory $\psi$ of example 1 and the new information $\mu$ of example 5. The elements of the set $\Gamma$ are given by the following table:*

| $D$ | $D_\psi \cap \overline{D_\mu}$ |
|---|---|
| $p_1 \wedge p_3 \wedge p_4$ | $\{\neg p_1, \neg p_3, p_4\} \cap \overline{\{p_1, p_3\}} = \{\neg p_1^{\{2\}}, \neg p_3^{\{0\}}\}$ |
| $\neg p_1 \wedge p_2 \wedge p_3$ | $\{\neg p_1, \neg p_2\} \cap \overline{\{p_2, p_3\}} = \{\neg p_2^{\{0,1\}}\}$ |
| $p_1 \wedge \neg p_2 \wedge p_3$ | $\{\neg p_1, \neg p_2\} \cap \overline{\{p_1, p_3\}} = \{\neg p_1^{\{2\}}\}$ |
| $\neg p_1 \wedge p_2 \wedge p_3 \wedge p_4$ | $\{\neg p_1, \neg p_3, p_4\} \cap \overline{\{p_2, p_3\}} = \{\neg p_3^{\{0\}}\}$ |

*The first term is always eliminated, because it has $k_D = 2$ and the literals to be deleted represent two clauses in $PI_\psi$ (clauses 0 and 2). All the remaining terms have $k_D = 1$ and would be chosen by Dalal's algorithm, resulting in the same revised belief base that was calculated in example 5:*

$$(\neg p_1 \wedge p_2 \wedge p_3) \vee (p_1 \wedge \neg p_2 \wedge p_3)$$

*that correspond to the following set of prime implicates: $(\neg p_1 \vee \neg p_2) \wedge (p_1 \vee p_2) \wedge p_3$.*

*But, if we take into account the size of the conjunctive coordinate sets of the literals to be deleted, then only the last two terms would be chosen and the resulting revised belief base would be:*

$$(\neg p_1 \wedge p_2 \wedge p_3 \wedge p_4) \vee (p_1 \wedge \neg p_2 \wedge p_3)$$

*that correspond to the following set of prime implicates: $(p_1 \vee p_4) \wedge (\neg p_2 \vee p_4) \wedge (\neg p_1 \vee \neg p_2) \wedge (p_1 \vee p_2) \wedge p_3$.*

*For a belief base $\neg p_1 \wedge \neg p_2 \wedge (\neg p_3 \vee \neg p_2)$, i.e., the same as $\psi$ but without $p_4$, Dalal's method returns the same result. In this case, the proposed method returns $\psi \widehat{\circ} \mu = p_1 \wedge \neg p_2 \wedge p_3$ that correspond to the following set of prime implicates: $p_1 \wedge \neg p_2 \wedge p_3$.*

*It can be seen that, as expected, the proposed method preserves one original clause in both cases: clause $\neg p_2 \wedge p_4$ for $\psi$ and clause $\neg p_2$ for $\psi$ without $p_4$. In both cases, Dalal's method does not preserve any clause.*

The status of the proposed method with respect to the AGM postulates is given by the following theorem[6]:

[6] The proofs of the theorems 2 and 3 are presented in [4].

**Theorem 2** *The revision operator $\widehat{\circ}$ satisfies* **R1**$\sim$**R6**, *as defined in Section 4.1.*

Both procedures, the one that calculates the revised belief base according to Dalal's definition, based on $k_D$, and the new proposed procedure, that uses $\widehat{k}_D$, assume that the belief base $\psi$ is represented by $IP_\psi$. An interesting property of the new proposed revision method is that the revised belief base can also be determined from $PI_\psi$. Consider as before that new information $\mu$ is given by $IP_\mu$. Initially, for each $D_\mu \in IP_\mu$, we calculate the set of clauses that are inconsistent with $D_\mu$:

$$Clash(D_\mu) = \{C \mid C \in PI_\psi \text{ and } C \subseteq \overline{D_\mu}\}$$

The DNF of the revised belief base is given by the prime implicants of the theory whose CNF is given by $PI_\psi - Clash(D_\mu)$ extended with unitary clauses containing the literals in $D_\mu$, for those $D_\mu$ associated with the sets $Clash(D_\mu)$ with minimum size. More formally, let $Min = \{D_\mu \mid D_\mu \in IP_\mu$ and $\mid Clash(D_\mu) \mid$ is minimal$\}$, the revised base is given by:

$$DNF_{\psi\widehat{\circ}\mu} = \bigcup_{D_\mu \in Min} DT(PI_\psi - Clash(D_\mu) \cup \{\{L\} \mid L \in D_\mu\}) \tag{2}$$

where $DT$ is the dual transformation described in Section 3.

**Theorem 3** *The revised belief base obtained by equation 1 is equivalent to the revised belief base obtained by equation 2.*

Although this method for determining $DNF_{\psi\widehat{\circ}\mu}$ is potentially much more expensive than the previously presented one, the fact that the result is exactly the same shows that the proposed revision really deletes the minimum number of prime implicates of $\psi$ in such a way that the resulting theory is consistent with $\mu$.

## 6 CONCLUSION

This paper presented a new syntactical method to calculate the belief revision operator introduced by Dalal [6] that requires only one NP-complete calculation instead of multiple NP-complete calculations. The method is based on a special representation for the prime implicants/implicates normal forms, called the quantum notation, and an algorithm to calculate this representation, given a CNF or DNF normal form, is also introduced.

The paper also introduced a new belief revision operator, based on the idea that one clause in the unique prime implicate normal form is a better candidate for a minimal information chunk than the one propositional truth value change, the semantic option chosen by Dalal.

The algorithms presented in the paper have been implemented in Common Lisp [22] and tested with the theories in the SATLIB (`http://www.satlib.org/`) benchmark.

In our future work, we plan to apply our approach to belief update. Namely, we want to revisit belief update operators such as the operator proposed by Forbus [10].

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