

Plausibility structures for default reasoning

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Abstract. Friedman and Halpern have introduced the inference by plausibility structures, which provides semantics for various default logics. This is a generalization of known inferences, such as those based on expectations, or on possibilistic logic. We argue that a slightly different inference by plausibility would be more appropriate for default reasoning. It would still be appropriate for all the default logics considered by Friedman and Halpern. Significant cases, such as subsets of normal defaults with multiple extensions, or a formalism extending circumscription to the cases where (AND) is falsified, can be translated into the new formalism only.

In order to prove our results, we complete the list of the reasoning properties for the (two versions of the) inference by plausibility. Since these properties describe the behavior of a given inference in an intuitive and non technical way, this list is important for any potential user. Moreover, it happens that considering these properties only, without going into the technicalities of the plausibility approach, is enough to describe the main results given here.

1 INTRODUCTION

Friedman and Halpern have introduced *inference by plausibility* and shown it to encompass various kinds of default reasoning [4, 5]. A feature of this inference is that the rule (AND) – if ψ_1 and ψ_2 can be derived, then their conjunction $\psi_1 \wedge \psi_2$ can be derived – is not necessarily satisfied. Many interesting cases where the rule (AND) should not be satisfied are known (e.g. when using defaults for expressing expected results of *indeterministic actions*, a common application of default reasoning). However, against the main traditions in default reasoning, the inference by plausibility defined in [4, 5] must satisfy a special case of the rule (AND), where ψ_1 and ψ_2 are contradictory. As a consequence, with any subset of the *normal defaults* of Reiter [10] (the simplest and the most often used defaults in Reiter’s tradition), this enforces the rule (AND) in full generality.

Notice that Friedman and Halpern have not provided any example of default formalism without (AND). Moreover, they have detected an embarrassing behavior even when (AND) is satisfied. We propose a slight modification of the original definition. We describe the reasoning properties (those that any potential user should know) of the new version, and of the original one. Thanks to these reasoning properties, we prove the following results about the modified formalism with respect to the original version:

1. The main unexpected or unwanted behaviors, either in the general case or in the case where (AND) holds, are suppressed.
2. All the default formalisms considered by Friedman and Halpern can also be translated into the new version.
3. There exist useful known formalisms for default reasoning that can be translated into the new version only.

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In section 2 we introduce the logical framework, and Friedman and Halpern’s inference by plausibility, exhibiting unwanted properties of this inference in section 3. In section 4 we introduce an alternative definition, simpler and more powerful than the original version, completing the comparison in section 5. In section 6 we examine a known default formalism which can be translated into the new version only. In section 7 we give the necessary and sufficient conditions which allow one inference by plausibility to be translated into the other version. These results show that all the default formalisms considered by Friedman and Halpern can also be translated into the new version.

We omit the proofs which do not present real difficulty, however we take care of providing enough intermediate results in order to make each isolated proof rather easy to reconstruct.

2 INFERENCES FOR DEFAULT REASONING

Since Reiter [10], a great number of formalisms have been considered for inferring conclusions *by default*, meaning from what is presently known: inferences “in the absence of other information”. In this text, for the sake of simplicity, we consider only the propositional case, and inferences from a single formula. Let us denote by \mathbf{L} the set of all the classical formulas, and by \Rightarrow the classical implication. We introduce a *default connector* \rightarrow . Informally, for any φ, ψ in \mathbf{L} , $\varphi \rightarrow \psi$ means “if φ , then typically ψ ”. Since classical way of reasoning is assumed, we will consider only the formalisms satisfying the reasoning properties of “left logical equivalence” (LLE), “right weakening” (RW) and “reflexivity” (REF) as defined by Kraus, Lehmann and Magidor in [7]:

- (LLE) If $\vdash_{\mathbf{L}} \varphi \Leftrightarrow \varphi'$, then from $\varphi \rightarrow \psi$, we get $\varphi' \rightarrow \psi$.
- (RW) If $\vdash_{\mathbf{L}} \psi \Rightarrow \psi'$, then from $\varphi \rightarrow \psi$, we get $\varphi \rightarrow \psi'$.
- (REF) $\varphi \rightarrow \varphi$.

An equivalent way for describing a default inference is by defining a mapping f from \mathbf{L} to the set of the sets of formulas $\mathcal{P}(\mathbf{L})$:
 $\psi \in f(\varphi)$ iff $\varphi \rightarrow \psi$, i.e., $f(\varphi) = \{\psi \in \mathbf{L} / \varphi \rightarrow \psi\}$.

This text will use two kinds of inference by plausibility, the two mappings being denoted by f_{FH} and f_N , and the two default connectors by \xrightarrow{FH} and \xrightarrow{N} .

(RW) implies (RLE), “right logical equivalence”: if $\vdash_{\mathbf{L}} \psi \Leftrightarrow \psi'$, then from $\varphi \rightarrow \psi$, we get $\varphi \rightarrow \psi'$. (LLE) and (RW) being required, we consider that \mathbf{L} is the set of all the equivalence classes of formulas, identifying a formula with its equivalence class. The “true” and the “false” formulas are defined as follows: **true** = $\varphi \vee \neg\varphi$, **false** = $\varphi \wedge \neg\varphi$.

Identifying a formula with its equivalence class concerns only (LLE) and (RLE), but not full (RW). In terms of f , (RW) is: If $\vdash_{\mathbf{L}} \psi \Rightarrow \psi'$ and $\psi \in f(\varphi)$, then $\psi' \in f(\varphi)$. This means that the

set $f(\varphi)$ is not any subset of \mathbf{L} , but a union of theories, called here a *U-theory*, where a *theory* \mathcal{T} is a set of formulas closed by deduction: $\mathcal{T} = \{\varphi \in \mathbf{L}/\mathcal{T} \vdash_{\mathbf{L}} \varphi\}$. Let us denote by \mathbf{T} the set of all the theories in \mathbf{L} and by \mathbf{U} the set of all the U-theories. Clearly $\mathbf{T} \subseteq \mathbf{U}$.

Thus, f is a mapping from \mathbf{L} to \mathbf{U} . Restricting our attention to these mappings f means precisely restricting our attention to connectors \rightarrow satisfying (LLE) and (RW). If moreover we require

$$\varphi \in f(\varphi) \text{ for each } \varphi \in \mathbf{L},$$

we take also (REF) into account. Since Reiter's proposal (where the set of all the extensions is indeed a U-theory), a stronger reflexivity (sREF) is implicitly assumed in default reasoning [if we conclude $Flies(Tweety)$ from $Bird(Tweety)$, we want to conclude also $Bird(Tweety) \wedge Flies(Tweety)$ from $Bird(Tweety)$]:

(sREF) If $\psi \in f(\varphi)$, then $\varphi \wedge \psi \in f(\varphi)$, i.e., if $\varphi \rightarrow \psi$, then $\varphi \rightarrow \varphi \wedge \psi$.

Sometimes, we need also the rules (Deduc) ("deduction"), (AND), (OR), ("case reasoning"), (CM) ("cumulative monotony"), (rCM) ("restricted (CM)"), and (CT) ("cumulative transitivity"):

- (Deduc)** If $\varphi \wedge \psi \rightarrow \psi'$, then $\varphi \rightarrow \psi \Rightarrow \psi'$.
- (AND)** If $\varphi \rightarrow \psi$ and $\varphi \rightarrow \psi'$, then $\varphi \rightarrow \psi \wedge \psi'$.
- (OR)** If $\varphi \rightarrow \psi$ and $\varphi' \rightarrow \psi$, then $\varphi \vee \varphi' \rightarrow \psi$.
- (CM)** If $\varphi \rightarrow \psi$ and $\varphi \rightarrow \psi'$, then $\varphi \wedge \psi \rightarrow \psi'$.
- (rCM)** If $\vdash_{\mathbf{L}} \psi' \Rightarrow \psi$ and $\varphi \rightarrow \psi'$ then $\varphi \wedge \psi \rightarrow \psi'$.
- (CT)** If $\varphi \rightarrow \psi$ and $\varphi \wedge \psi \rightarrow \psi'$, then $\varphi \rightarrow \psi'$.

Here are two translations in terms of the mapping f . Recall that f satisfies (LLE), (RW) and (REF), i.e. that f is a mapping from \mathbf{L} to \mathbf{U} satisfying $\varphi \in f(\varphi)$: (AND) f is a mapping from \mathbf{L} to \mathbf{T} .
(OR) $f(\varphi) \cap f(\varphi') \subseteq f(\varphi \vee \varphi')$.

Most of these properties are well known. However, (CM) and (REF) are est motivated when (AND) holds. Since (AND) does not always hold, it is useful to introduce the variations (sREF) and (rCM). Clearly, (AND) and (REF) imply (sREF), (CM) implies (rCM), (sREF) implies (REF) (recall that (RW) is assumed here).

Definition 2.1 A *d-mapping* ("d" standing for "default") is a mapping $f : \mathbf{L} \rightsquigarrow \mathbf{U}$ which satisfies (sREF).

A d-mapping is an inference satisfying (LLE), (RW) and (sREF). In this "core of hardly avoidable properties" for default reasoning, we exclude (AND), (Deduc), (rCM), and (CT), falsified by some formulations of Reiter's defaults. Reiter's defaults in their two formulations ("skeptical" when considering the intersection of all the extensions, versus standard or "brave"), and most of its variants, can be studied as d-mappings (satisfying (AND) for the skeptical formulations only). For d-mappings satisfying (AND), (rCM) and (CM) are equivalent, and so are (Deduc) and (OR). For ordinary d-mappings, we get only that (CM) implies (rCM) and that (OR) implies (Deduc).

A *basic inference relation* \mathcal{B} of Bochman [2] is a d-mapping satisfying (Deduc) and (rCM). Bochman implicitly uses the concept of d-mapping, when describing results true "in the framework of \mathcal{B} ", such as the equivalence between (Deduc) and (Confirmation):

(Confirmation) If $\varphi \rightarrow \psi$ then $\varphi \vee \psi' \rightarrow \psi \vee \psi'$.

It seems natural to give a stronger importance to properties true in almost any default formalisms, such as the properties of d-mappings, than to properties falsified by various leading default formalisms (as in [10]), such as (Deduc) or (rCM).

Proposition 2.2 A *d-mapping* can be defined by the set of ordered couples (φ, φ') such that $\varphi \wedge \varphi' \vdash_{\mathbf{L}} \text{false}$ and $\varphi \vee \varphi' \rightarrow \varphi$.

Indeed, by (LLE), (sREF) and (RW) (notice that (AND) is not required here), we have $\varphi \rightarrow \psi$ iff $(\varphi \wedge \psi) \vee (\varphi \wedge \neg \psi) \rightarrow \varphi \wedge \psi$.

This result has been more or less implicitly used by various approaches for default reasoning, such as "expectations" [6] or possibilistic logic [1], and by Friedman and Halpern which, in [4, 5], have generalized these approaches as described now. In particular, their generalization allows for the possibility that (AND) does not hold (at least in the definitions, if not in the defaults considered), unlike the definitions of [6, 1]:

A *plausibility space* is a tuple $(W, \mathbf{F}, Pl, D, \leq)$ where W and D are sets, \mathbf{F} is a subset of the set $\mathcal{P}(W)$ of the subsets of W which is closed under union and complementation with respect to W , Pl is a mapping from \mathbf{F} to D , the set D being partially ordered by the relation \leq . The strict order relation $<$ is defined as usual by $d_1 < d_2$ if $[d_1 \leq d_2$ and $d_2 \not\leq d_1]$ i.e., if $[d_1 \leq d_2$ and $d_1 \neq d_2]$. Pl must satisfy condition (A1):

$$(A1) \quad \text{If } A \subseteq B \text{ then } Pl(A) \leq Pl(B).$$

Thus, denoting $Pl(\emptyset)$ by \perp and $Pl(W)$ by \top , we get, for each $d \in Pl(\mathbf{F})$: $\perp \leq d \leq \top$.

A *plausibility structure* PL in \mathbf{L} is a plausibility space where the set of *worlds* W is the set M of all the classical interpretations for \mathbf{L} . For any formula $\varphi \in \mathbf{L}$, $[[\varphi]]$ denotes the set of the elements of $W (= M)$ that satisfy φ : $[[\varphi]] = \{w \in W/w \models_{\mathbf{L}} \varphi\}$. \mathbf{F} is the set $\{[[\varphi]]/\varphi \in \mathbf{L}\}$ of all the subsets $[[\varphi]]$ of W .

The original definition in [4, 5] is apparently more general, since W is any set of "copies of elements" of M . The notion of inference by plausibility is unmodified by our simplifications. It follows that Pl could be defined as a mapping from \mathbf{L} to D : we can identify $Pl(\varphi)$ with $Pl([[\varphi]])$. Here is the writing of (A1) then:

$$(A1) \quad \text{If } \vdash_{\mathbf{L}} \varphi \Rightarrow \psi, \text{ then } Pl(\varphi) \leq Pl(\psi).$$

Definition 2.3 [4, 5] The *inference by plausibility* is then defined as follows: given a plausibility structure PL , we define

$$PL \models \varphi \xrightarrow{FH} \psi \text{ if either } Pl(\varphi) = \perp \text{ or } Pl(\varphi \wedge \psi) > Pl(\varphi \wedge \neg \psi).$$

We define the mapping f_{FH} from \mathbf{L} to $\mathcal{P}(\mathbf{L})$, associated to PL :

$$f_{FH}(\varphi) = \{\psi \in \mathbf{L}/Pl \models \varphi \xrightarrow{FH} \psi\}.$$

Proposition 2.4 The *inference by plausibility* f_{FH} is a *d-mapping* satisfying (rCM).

The original texts give (LLE) and (RW), while (sREF) and (rCM) are immediate. This shows that the inference by plausibility is indeed well fitted for default reasoning, since it is a d-mapping.

In order to catch various kinds of default formalisms, we can expect that all we need is some additional properties on Pl . Friedman and Halpern show that this is the case for some rules such as (AND) and (OR) and for the system \mathbf{P} of [7]. The system \mathbf{P} is defined as respecting the rules (AND), (OR) and (CM) [together with (LLE), (RW) and (sREF) which, in case of (AND) and (RW), is equivalent to (REF)]. This system satisfies also (CT). System \mathbf{P} was already known to have many possible apparently different definitions or semantics (we refer the reader to [5] for more on this subject).

The inference by plausibility has however a strange behavior, noticed in [5]: in the case of (AND), the inference satisfies (OR), except possibly for the “false case” $Pl(\varphi) = Pl(\varphi') = \perp$, called (fOR) here. This particular case is so bewildering that [5] never considers (AND) in isolation, but only the combination (AND) + (fOR). For what concerns default reasoning in the absence of (AND), [5] does not describe any already known system. At least since Reiter [10], the non (AND) case is however known to be inescapable in many cases of default reasoning (consider e.g. actions with indeterministic effects). In this text, we examine also the non (AND) case.

3 NOT SO DESIRABLE PROPERTIES

Since for any d-mapping, (rCM) and (AND) imply (CM), we get the the result, considered as surprising in [5], that, in the presence of (AND), an inference by plausibility satisfies also (CM).

Apart from the (OR) problem in case of (AND), it could seem that the inference by plausibility is a good candidate for providing a “semantics” for default reasoning. Indeed, (rCM), even if not satisfied by all the main previous formalisms for default reasoning (see Reiter’s “skeptical” counter-example), is mild enough to be accepted as a good reasoning principle. From *birds fly* and *flying animals are always light*, it seems natural to conclude *light birds fly*.

We consider now the case where (AND) is not required. Here are the “false cases” of two properties already introduced:

- (fDeduc) If $\varphi \wedge \psi \rightarrow \text{false}$ then $\varphi \rightarrow \neg\psi$.
- (fAND) If $\varphi \rightarrow \psi$ and $\varphi \rightarrow \neg\psi$, then $\varphi \rightarrow \text{false}$.

Propositions 3.1 1. *The inference by plausibility f_{FH} satisfies (Deduc), except possibly the false case (fDeduc).*

2. *Any inference by plausibility f_{FH} satisfies (fAND).*

“Near (Deduc)” without full (Deduc), and also (fAND) even in the absence of full (AND), are hard to justify from a reasoning point of view. Any potential user must be aware of these facts. For the second point, recall that for *normal defaults* [10], (fAND) implies (AND). Another problem with (fAND) is that, contrarily to (AND) or (OR), it does not generalize fully to more than two formulas. Indeed, we may have $\varphi \xrightarrow{FH} \psi_i, i \in \{1, \dots, 3\}, \bigwedge_{i=1}^3 \psi_i \vdash_{\mathbf{L}} \text{false}$, without getting $\varphi \xrightarrow{FH} \text{false}$. Thus, (fAND) means that f_{FH} is a mapping from \mathbf{T} to the bizarre subset of \mathbf{U} consisting of the unions of theories such as these unions are not contradictory in the precise case when there are exactly two distinct theories in the union. Such restrictions could be justified only if there were good computability reasons.

Notice that the problem with (fAND) does not concern the already quoted formalisms of [6, 1] since (AND) holds there.

4 ANOTHER INFERENCE BY PLAUSIBILITY

Let us simplify the definition. We will show that doing this, we define a slightly different notion which has a better set of intuitive properties of reasoning, and, for what concerns default reasoning, is more powerful and encompasses more already known formalisms.

Definition 4.1 *The inference by N-plausibility (N for “new”) is defined as follows: given a plausibility structure PL , we define*

$PL \models \varphi \xrightarrow{N} \psi$ if $Pl(\varphi \wedge \psi) \geq Pl(\varphi \wedge \neg\psi)$, and $f_N : \mathbf{L} \rightsquigarrow \mathcal{P}(\mathbf{L})$ by $f_N(\varphi) = \{\psi \in \mathbf{L} / PL \models \varphi \xrightarrow{N} \psi\}$.

Theorem 4.2 *The inference f_N is a d-mapping satisfying (Deduc) and (rCM).*

This means that, contrarily to f_{FH} , f_N is a “basic inference relation” as defined in [2]. Moreover, in contrast with the bewildering behavior of f_{FH} , with f_N we get: (AND) implies (CT), which implies (OR) (Theorem 4.4 below), while (fAND) is not mandatory.

We need now four properties that a plausibility measure may possess: A, B and C are subsets of W (in \mathbf{F}) which are pairwise disjoint.

- (A2_N) If $Pl(A \cup B) \geq Pl(C)$ and $Pl(A \cup C) \geq Pl(B)$,
then $Pl(A) \geq Pl(B \cup C)$.
- (A3) If $Pl(A) = Pl(B) = \perp$, then $Pl(A \cup B) = \perp$.
- (A6_N) If $Pl(A \cup B) \geq Pl(C)$ and $Pl(A) \geq Pl(B)$,
then $Pl(A) \geq Pl(B \cup C)$.
- (A7_N) If $Pl(A) \geq Pl(C)$ and $Pl(A) \geq Pl(B)$,
then $Pl(A) \geq Pl(B \cup C)$.

(A7_N) implies (A3) [$A = \emptyset$ in (A7_N)] and, thanks to (A1), (A2_N) implies (A6_N) which implies (A7_N). If the plausibility is a probability, (A3) is satisfied. We get the following results, where the already evoked *false case* (“trivial case” in [5]) of (OR) is defined as follows:

(fOR) If $\varphi_1 \rightarrow \text{false}$ and $\varphi_2 \rightarrow \text{false}$ then $\varphi_1 \vee \varphi_2 \rightarrow \text{false}$.

Propositions 4.3 1. *An inference f_N by N-plausibility satisfies (AND) iff Pl satisfies (A2_N).*

2. *An inference f_N by plausibility satisfies (fOR) iff Pl satisfies (A3).*

3. *f_N satisfies (CT) iff Pl satisfies (A6_N).*

4. *f_N satisfies (OR) iff Pl satisfies (A7_N).*

Thus, the behavior is natural and easy to grasp for any user:

Theorem 4.4 *If an inference by N-plausibility f_N satisfies (AND), then it satisfies (CT) [true for any d-mapping satisfying (Deduc)]. If f_N satisfies (CT), then it satisfies (OR). These implications are strict.*

Notice that, exactly as with the original definition \xrightarrow{FH} , by (A1) we get: $\varphi \xrightarrow{N} \text{false}$ iff $Pl(\varphi) = \perp$.

5 COMPARING THE TWO VERSIONS

We need three other properties of a plausibility measure.

Let A, B and C be three pairwise disjoint subsets in \mathbf{F} .

- (A2) If $Pl(A \cup B) > Pl(C)$ and $Pl(A \cup C) > Pl(B)$,
then $Pl(A) > Pl(B \cup C)$.
- (A6) If $Pl(A \cup B) > Pl(C)$ and $Pl(A) > Pl(B)$,
then $Pl(A) > Pl(B \cup C)$.
- (A7) If $Pl(A) > Pl(C)$ and $Pl(A) > Pl(B)$,
then $Pl(A) > Pl(B \cup C)$.

Friedman and Halpern have introduced (A1), (A2) and (A3) in [4, 5], which explains the absence of (A4) and (A5), introduced in [4, 5] for properties not considered here. (A2) implies (A6) which implies (A7), but (A7) does not imply (A3).

Propositions 5.1 1. *f_{FH} satisfies (AND) iff Pl satisfies (A2).*

2. *f_{FH} satisfies (fOR) iff f_{FH} satisfies (Deduc) iff Pl satisfies (A3).*

3. *f_{FH} satisfies (CT) iff Pl satisfies (A6).*

4. *f_{FH} satisfies (OR) iff Pl satisfies (A7) and (A3).*

These results either appear in [5], or are immediate, and they give the following important results about the reasoning properties :

Theorem 5.2 *If f_{FH} satisfies (AND), then it satisfies (CT) [5]. If f_{FH} satisfies (CT) and (Deduc), then it satisfies (OR). These implications are strict.*

Recall also that (OR) implies (Deduc) for any d-mapping.

The following property of a plausibility measure plays a crucial role for f_N . Let A and B be any disjoint elements of \mathbf{F} .

(A0) If $Pl(A) = Pl(B)$, we get $Pl(A \cup B) = \perp$.

(A0) implies (A3), but (A0) is clearly much stronger than (A3).

Proposition 5.3 *An inference by N-plausibility f_N satisfies (fAND) iff Pl satisfies (A0). Thus, if an inference by N-plausibility f_N satisfies (fAND), it must satisfy (fOR).*

Obviously, (A2_N) implies (A0) [take $A = \emptyset$ in (A2_N)] which, for f_N , corresponds to the equally obvious fact that (AND) implies (fAND).

Notice that, since (A6_N) does not imply (A0), an inference by N-plausibility can satisfy (CT) and falsify (fAND).

Let us summarize now why we think that the new way f_N of using plausibility structures for defining a default inference is better than the original way f_{FH} :

(1) The property (Deduc) is satisfied by f_{FH} , except possibly when $A \wedge B$ has a null plausibility. Thus, f_N is a “basic inference relation” as defined in [2] while this is not the case for f_{FH} .

(2) The bewildering property (fAND) is mandatory with f_{FH} . Except in cases where full (AND) holds, this makes the reasoning behavior of f_{FH} rather complex to explain to a potential user. With f_N , (fAND) has a semantical counterpart (A0), a strong requirement, in accordance with the strength of (fAND).

(3) If now we consider the case where (AND) holds, here again f_N is better than f_{FH} , since f_N does not have the bewildering property “(OR) except for formulas with a null plausibility”.

Still, we cannot encompass by f_N all the already known default formalisms: in Reiter’s original proposal e.g., (AND) does not imply (OR) (since it does not imply (fOR) either, f_{FH} is not better than f_N here). However, we could expect to encompass at least the already known formalisms which respect the main reasoning properties exhibited by the inferences by plausibility, such as “(AND) implies (CM)” [cf. (rCM)] and “(Deduc)+(CT) implies (OR)”. Let us examine one such candidate now.

6 THE CASE OF X-logic

Definition 6.1 [11] For any $X \subseteq \mathbf{L}$, we define the *X-mapping* f_X from \mathbf{L} to $\mathcal{P}(\mathbf{L})$ by: $\varphi \in f_X(\psi)$ iff $Th(\psi \wedge \varphi) \cap X \subseteq Th(\psi)$.

We use the notation $\psi \rightarrow_X \varphi$ for $\varphi \in f_X(\psi)$.

This notion had been used by Suchenek [12] (referring himself to one the earliest works on effective computation for a non-monotonic formalism [3]) in order to help the computation of circumscription and related formalisms. Circumscription satisfies (AND), thus Suchenek was only concerned by the cases satisfying (AND), however, generally, X-mappings falsify (AND) [11]. The notion of X-mapping remains as one of the most promising ways for introducing an efficiently computable default inference without (AND). Each finite circumscription is an X-logic, which thus also provides an interesting alternative way for effective computation of circumscription. The next results summarize the importance of X-mappings from a default reasoning perspective, even in the absence of (AND).

Theorem 6.2 [11, 9, 2] *Any X-mapping is a d-mapping satisfying (CT), (OR) and (rCM). Conversely, in the finite case, any d-mapping satisfying (CT), (OR) and (rCM) is an X-mapping.*

Thus, X-mappings look like very promising candidates to try in order to sustain the claim that inferences by plausibility encompass various default formalisms. Disappointingly, generally X-logics are not inferences by plausibility f_{FH} . This is only due to the fact that, as expected for a reasonable default formalism, X-logics falsify (fAND). The next result illustrates the power and the convenience of f_N :

Proposition 6.3 *Any X-mapping, defined by a set X , is equal to the inference f_N defined by the following plausibility structure:*

We define the plausibility Pl_X : for each $\varphi \in \mathbf{L}$, $Pl_X(\varphi) = \{x \in X/\varphi \vdash_{\mathbf{L}} x\}$. The set D is the set $\{Pl_X(\varphi)/\varphi \in \mathbf{L}\}$ and the partial order \geq is \subseteq , the set inclusion on D (i.e., \leq is \supseteq).

7 FROM f_N TO f_{FH} AND BACK

It is enough to consider a few natural reasoning properties, without going inside the technicalities of plausibility structures, in order to state the main results (not their proofs however).

Theorem 7.1 *An inference by N-plausibility f_N is equal to an inference by plausibility f_{FH} iff f_N satisfies (fAND).*

Proof: See Proposition 3.1-2 if f_N falsifies (fAND). If f_N satisfies (fAND), $\varphi \xrightarrow{N} \psi$ iff $Pl(\varphi \wedge \psi) \geq Pl(\varphi \wedge \neg\psi)$ iff $[Pl(\varphi \wedge \psi) > Pl(\varphi \wedge \neg\psi)$ or $(\varphi \xrightarrow{N} \psi$ and $\varphi \xrightarrow{N} \neg\psi)]$ iff, by (fAND) $[Pl(\varphi \wedge \psi) > Pl(\varphi \wedge \neg\psi)$ or $\varphi \xrightarrow{N} \text{false}]$ iff $[Pl(\varphi \wedge \psi) > Pl(\varphi \wedge \neg\psi)$ or $Pl(\varphi) = \perp]$. \square

So, when this passage is possible, it is obvious: the plausibility structure is unmodified. This result also shows that any “reasonable” f_{FH} is an f_N : the only problem being (fAND), of arguable cognitive interest. The passage from the original definition to the new one is not so obvious, but it remains rather easy also:

Theorem 7.2 *An inference by plausibility f_{FH} is equal to an inference f_N iff f_{FH} satisfies (fOR) [or equivalently (Deduc)].*

Recall that, for f_{FH} and for f_N , the inference satisfies (fOR) iff PL satisfies (A3). Since this result and its proof are important, and since this is the only result of this text for which the proof is not quasi automatic, here are the proof of the easy way and the useful intuitions for the (too long to appear here in full length) proof of the other way.

\Rightarrow : This proof uses a “restricted false case of (OR)” (rfOR). Even if this is not directly related to the proof, it can be interesting, in order to get a more precise view of the reasoning properties of the inferences by plausibility, to give here also two kinds of “weak relative consistency”, (P) and its strengthening (P’).

- (rfOR) If $\varphi_1 \rightarrow \text{false}$ and $\varphi_2 \rightarrow \text{false}$, then $\varphi_1 \vee \varphi_2 \rightarrow \varphi_1$.
- (P) If $\varphi \rightarrow \text{false}$ and $\psi \rightarrow \varphi$, then $\psi \rightarrow \neg\varphi$.
- (P’) If $\varphi \rightarrow \text{false}$ and $\psi \rightarrow \varphi$, then $\psi \rightarrow \text{false}$.

The following lemma is immediate:

- Lemma 7.3** 1. f_{FH} and f_N satisfy (P).
- 2. (P’) is satisfied by f_{FH} and is equivalent to (fOR) for f_N .
- 3. f_N satisfies (rfOR).
- 4. f_{FH} satisfies (rfOR) iff it satisfies (fOR).

Notice in passing that for f_N and for f_{FH} , property (P) shows that (rfOR) is in fact equivalent to its apparent strengthening (rfOR’):

(rfOR') If $\varphi_i \rightarrow \text{false}$ for $i \in \{1, 2\}$, then $\varphi_1 \vee \varphi_2 \rightarrow \varphi_1$ and $\varphi_1 \vee \varphi_2 \rightarrow \neg \varphi_1$.

Here is the proof of the \Rightarrow side of the theorem: If f_{FH} falsifies (fOR), it falsifies (rfOR), which is satisfied by each inference by N -plausibility, thus the translation is impossible.

\Leftarrow : (Main intuitions only here, see [8] for a proof in French) If f_{FH} satisfies (fOR), we introduce (Pl', \leq') from (Pl, \leq) defining f_{FH} . By (A3), we can keep the “equivalence class” (two formulas being “equivalent” iff they have the same plausibility) of the formula **false**. We must break the other “equivalence classes”. In order to simplify the definition of Pl' , we can break them into singletons. Here are the three conditions that must be satisfied by (Pl', \leq') : (1) two contradictory formulas in the same “class” for Pl (except the “null class”) must be incomparable for Pl' (they cannot stay in the same “class”, but this is not enough), (2) condition (A1), and (3), “otherwise preserve the relation” induced on \mathbf{L} by (Pl, \leq) . \square

Each time f_{FH} satisfies (fOR), i.e. is defined by a plausibility satisfying (A3), we can use f_N instead of f_{FH} . Thus, the results given in [5] for system P , and also the “direct semantical translations” for “parameterized probability distribution” and for preferential structures, apply to f_N also. Theorem 7.2 sustains our claim that all the default inferences considered in [5] could alternatively be translated into the new and simpler version proposed here. And we have shown why we think that the new version has a better behavior and encompasses more default formalisms than the original one.

Notice that if we require that \leq is (and remains) a linear order on D , this translation does not apply. So, this argument in favor of f_N against f_{FH} does not apply to the formalisms evoked above from [1, 6], but only to the generalization to a partial order made in [5].

Since (A0) is much stronger than (A3), if we count the inferences, the new version is “more powerful” than the original one. Indeed, the original one can be translated into the new one iff its plausibility satisfies the relatively weak condition (A3). The new version can be translated into the old one only when the stronger condition (A0) is satisfied. Thus, the number of inferences that fall prey to the new version is greater than the number of the inferences that fall prey to the original version (at least in the finite case).

Also, the “core” made of the inferences which can be expressed by either version is interesting in itself, since it contains e.g. all the default formalisms examined in [5].

8 CONCLUSION AND PERSPECTIVES

The present work has introduced a new and simpler notion of inference by plausibility, and has given a list of the main properties, in terms of reasoning, of these two kinds of inference. These properties are very important, since the first thing to do before trying a translation of a given formalism in terms of plausibility is to check, from these properties, whether a translation is possible or not. This is why we have given new properties for the original inference by plausibility together with the main properties satisfied by the new version. Thanks to these properties, we could describe the cases in which one kind of inference by plausibility can be translated into the other. These inter-translations show that the new version is more powerful than the original one, even if there is no inclusion.

The main interest of the new notion is a better “reasoning behavior”. Indeed, it does not enforce the “(AND) for contradictory formulas”, called (fAND) here, that the original formalization satisfies. This property is clearly “against the spirit of default reasoning”, as

shown by the case of normal defaults where (fAND) implies full (AND). Also, the new version satisfies the property of deduction, while the original version falsifies an important particular case of deduction. Moreover, the behavior of the new version is better in the case of (AND), as shown by the “false case of (OR)” problem with the original version.

The present work should enforce the interest of the notion of inference by plausibility, since it extends its range of applicability, for what concerns default reasoning. Indeed, working with one version or with the other is rather similar.

Since several previously known formalisms for default reasoning can be translated in a natural way into an inference by plausibility, we can get interesting results in this way. As a particular example, let us evoke the case of X-mappings, which constitute one of the easiest to compute default formalisms allowing to discard (AND). In the finite case, a condition on the set X is known to hold iff an X-mapping satisfies (AND) (condition (C1) in [9]). With the translation into an inference by plausibility (new version only here), we get an alternative condition, (A2_N). Even if it is not clear for us now, it could be that this condition is easier to check than (C1). Many results of this kind, difficult with the traditional formalisms, could be found, thanks to the method of (the new way of using) plausibility.

It remains to characterize fully these two kinds of inference by plausibility, and to examine more default formalisms.

The nature of the definition of inference by plausibility makes that once a formalism is translated in these terms, this could help the effective computation, and this point also must be investigated.

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