A Qualitative Theory for Shape Representation and Matching for Design.

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Abstract. A new method for qualitative shape recognition and matching applied to the recognition and matching of objects in designs is presented. The paper presents an ordering information approach to the qualitative description of shapes considering qualitatively their angles, relative side length, concavities and convexities of the boundary and colour. The shapes recognised are regular and non-regular closed polygons without or with holes. To describe shapes with holes, topological and qualitative spatial orientation aspects are considered in order to relate the hole with its container. Each object is described by a string containing its qualitative distinguish features, which is used to match the object against others. The paper also describes how this method can be used in industrial design by explaining an application which final main objective is the automatic and intelligent assembling of mosaic borders using robot arms in the ceramic industry. This qualitative method provides several advantages over traditional quantitative representations. The main advantages are the reduction of computational costs and the managing of uncertainty.

1 INTRODUCTION

Human beings rely on qualitative descriptions of shape in many of their daily activities. The shape of an object is the description of the properties of the boundary of the object. A single point has neither dimension nor shape, but a one-dimensional curve has a shape that can be described. Shape is perhaps one of the most important characteristic of and object, and particularly difficult to describe qualitatively. The definition and use of a theory for qualitative description of shape is important in computer vision, which up to now uses quantitative methods with a high computational cost. The use of a qualitative theory for shape description and recognition will increase the efficiency in vision recognition because the recognition of a shape or an environment will be carried out by looking for only the salient characteristics and not analysing each pixel of the image.

Most of the qualitative approaches to shape description can be classified as follows:

- Axial representations: approaches based on a description of the axes of an object, describing the shape qualitatively by reducing it to a "skeleton" or "axis". The "axis" is a planar arc reflecting some global or local symmetry or regularity within the shape. The shape can be generated from the axis by moving a geometric figure (named "generator") along the axis and sweeping out the boundary of the shape. The generator is a constant shape and keeps a specified point (i.e. its centre) but can change its size and its inclination with respect to the axis [12].
- Primitive-based representations: approaches where complex objects are described as combinations of more primitive and simple objects. Here we can distinguish two schemes:
  - Generalized cylinder and geon-based representations, which describe an object as a set of primitives plus a set of spatial connectivity relations among them ([6]).
  - Constructive representations, which describe an object as the Boolean combination of primitive point sets ([5]).
- Ordering- and Projection-based representation: in these approaches different aspects of the shape of an object are represented by either looking at it from different angles or by projecting it onto different axes ([11], [14], [13]).
- Topology and logic-based representations: these approaches rely on topology and/or logics representing shapes ([1], [2]).
- Cover-based representations: in these approaches the shape of an object is described by covering it with simple figures, as rectangles and spheres ([3]).

The theory proposed in this article can be classified as an ordering based representation due to the fact that the theory orders the vertices of the objects to give its description. The theory is applied for the recognition of the shape of tiles for the automatic assembling of ceramic mosaic borders by a robot arm. A qualitative approach is the most suitable one for this application in order to manage the uncertainty provided by the fact that two manufactured tiles are never exactly identical. Moreover, the theory for qualitative description defined considers qualitatively the relative length of the edges and the colour of the object as distinguish features. These aspects have not been considered in other approaches.

2 THE ORDERING INFORMATION APPROACH TO QUALITATIVE SHAPE REPRESENTATION AND MATCHING. BASIC ASSUMPTIONS

The 2-dimensional ordering relations are defined for points, as a consequence, shape description using ordering information will have to make use of some reference points. As reference points we understand that points which completely specify the boundary. For

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polygonal boundaries (in this case we work with polygons without or with holes) it is natural to choose the vertices as reference points.

The basic assumptions to the qualitative theory for shape representation are referred to the way in which the reference points are numbered. They are:

- The start of the reasoning process is always in the uppermost (left) corner (vertex) of the object.
- The vertices are numbered from the starting vertex in a counter clockwise way.
- The edges between two vertices are classified as concave or convex.
- The angles in each vertex are either right-angled, acute or obtuse.
- The relative length of each edge between three contiguous vertices is also defined using a relative length model for compared lengths.
- As the colour is a relevant characteristic in the case of mosaic tiles, the colour of the shape is stored as RGB colour and then in the matching process the colour is considered qualitatively using the Delta E distance between colours.
- To describe the objects with holes the topological concept of Completely Inside Inverse (CI_i) [10] is used , due to the fact that the hole, in the case of tiles, is always completely inside the boundary of the object. The orientation spatial reference system or cardinal reference system of Frank [7] is used in order to relate each hole with each object detailing the position of each hole inside the object.

2.1 The length model

The length model developed compares lengths of two consecutive edges of the object. As we compare length at least two lengths are available, and as a result we find that one length is *bigger*, *smaller than* or *equal* to the other. Therefore the reference system named Length Reference System (LRS) is defined by a set of qualitative lengths labels. Thus, we define the LRS as: LRS={smaller, equal, bigger}.

2.2 Cardinal reference system.

For the description of the orientation of a hole inside an object the Cardinal Reference System of Frank [7] is used. In Frank's approach the cardinal directions reference system is used. The Cardinal Reference System divides space into eight or more cones (which allows working with different levels of granularity) (figure 1).

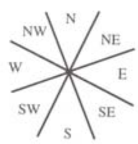


Figure 1. The cardinal directions reference systems with the space divided into eight angular regions.

2.3 The qualitative shape representation

2.4.1 The qualitative shape theory for objects without holes.

The qualitative shape representation theory is defined using as tool the Freksa and Zimmermann's orientation Reference System [9] augmented by a circle as it is described in [16]. In Freksa and Zimmermann's orientation model the space is divided into qualitative regions by means of a Reference System (RS). The RS is formed by an oriented line determined by two reference objects (from a point, *a*, to another point, *b*) –which defines the left/right dichotomy-, the perpendicular point by *b* –which defines the first front/back dichotomy-, and the perpendicular line by *a*- which establishes the second front/back dichotomy and defines a fine division of the space in the back part of the RS. If only the first/back dichotomy is considered (the RS is then called coarse) the space is divided into 9 qualitative regions. If both perpendicular lines are taken into account (the RS in then called fine) the space is divided into 15 qualitative regions. In our approach we focus our attention in the fine RS (figure 2a). An iconical representation of the fine RS and the names of the regions are shown in figure 2b). The information which can be represented by this RS is the qualitative orientation of a point object, *c*, with respect to (wrt) the RS formed by the point objects *a* and *b*, that is, *c wrt ab*.

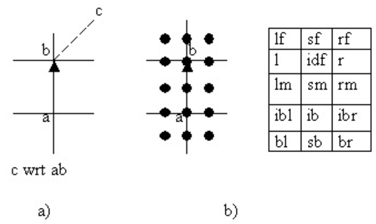


Figure 2. a) The fine RS for qualitative orientation; b) the iconical representation in which letters corresponds to l: left, r: right, f: front, s: straight, m: middle, b: back,, i: identical.

For the description of the angle type, we are going to use the augmented orientation Reference System by Zimmerman and Freksa [16]. This RS is defined by an oriented line from a to b, plus the two perpendicular lines by b and a respectively, and a circle with diameter ab (figure 3).

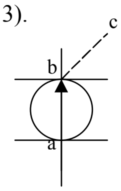


Figure 3. The fine orientation RS augmented by a circle with diameter ab.

The central idea of the qualitative shape representation consists in given three reference points i, j, k, which are consecutive due to the numeration given in a counter clockwise sense of the vertices of the object, the qualitative description of the reference point j is determined by positioning the orientation RS between the points i and k as figure 4 shows. In figure 4 i is the vertex 1, j is the vertex 2 and k is the vertex 3. In this figure the orientation RS is placed from 1 to 3.

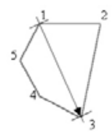


Figure 4. Example of a shape figure in which description of vertex 2 is determined using vertex 1 and 3 as the reference vertices for the orientation Freksa and Zimmermann's RS.

The qualitative description of the reference point j is given by a set of three elements (triple) <Aj,Cj,Lj> where Aj means the angle for the reference point j, Cj means the type of convexity of point j and

Lj means the relative length of the edges associated to reference point j (edge formed by vertices i and j versus edge formed by vertices j and k), where:

Aj $\in$ {right-angled, acute, obtuse};
Cj $\in$ {convex, concave} and
Lj belongs to the LRS defined in section 2.1.

The convexity of the point j is determined by the orientation RS as follows: if the reference point j remains on the left dichotomy created by the oriented line from i to k then the point j is a convex vertex. That means that the point j with respect to the Reference System formed by ik (j wrt ik) is on the left- front or left or left-medium or identical-back left or back-left. Otherwise if the point j remains on the right dichotomy created by the oriented line from i to k then the point j is concave, which means that j wrt ik is right-front or right or right-medium, or identical-back right or back-right. As a vertex appears when the orientation of the edge changes then it is not possible that the reference point j remains exactly over the oriented line from i to k. Formally, if Vj means vertex j we can formulate:

If Vj wrt ViVk $\in$ [lf, l, lm, ibl, bl] then Vj is convex.
If Vj wrt ViVk $\in$ [rf, r, rm, ibr, br] then Vj is concave.

The length calculated in the reference point j is the length of the edge from the point i to the point j compared with the length of the edge from the point j to k, using the LRS. Therefore it is inferred as:

- First the length of each edge is calculated using the Euclidean distance between two points: $D(V_i,V_j)=((X_{vj}-X_{vi})^2+(Y_{vj}-Y_{vi})^2)^{1/2}$
- Then, both lengths are compared and the corresponding label of the LRS is assigned as the value of the relative length to the vertex j.

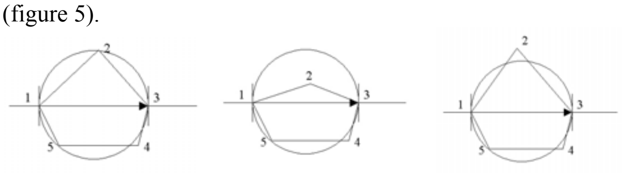
Finally the qualitative description of an angle is determined using the augmented orientation RS and some topological concepts as boundary, interior and exterior of an entity h, denoted as δh, h^o, h^- respectively.

If the reference point j remains exactly in the boundary of the circle of the augmented orientation RS, then the vertex j is right-angled, if j remains in the exterior of the circle then j is acute and if j remains in the interior of the circle then the vertex j is obtuse. Formally, if the circle of the augmented orientation RS with a diameter of ViVk is denoted as Cik, then the angle of the Vertex j (Vj) is calculated using the following algorithm:

If $Vj \cap \delta Cik \neq \varnothing$ then Vj is right- angled,
Else if $Vj \cap Cik^o \neq \varnothing$ then Vj is obtuse
Otherwise Vj is acute.

The part of the "otherwise" of the above algorithm occurs when $Vj \cap Cik^- \neq \varnothing$.

Next figure shows a graphical example for each of these cases (figure 5).



a) Right-angled angle b) Obtuse angle c) Acute Angle

Figure 5. Examples of determining the angle of vertex 2, using the augmented RS formed with vertices 1 and 3 as reference vertices.

2.4.2 The qualitative shape theory for objects with holes.

For describing an object with holes we follow next steps:

1. The qualitative shape description of the exterior boundary of the object (container) is constructed following the steps described in previous section.
2. Then the qualitative shape description of the boundary of each hole is constructed.
3. The holes and the container are related by adding two types of information:

3.1 First of all the topological relation between the container and the holes is fixed. In this case the topological relation chosen is the topological relation Completely Insidei (CI_i) defined in [10], because the holes, in the case of tiles, are always completely inside the container.

3.2 Secondly the orientation of each hole inside the container is determined (this is necessary because we can have an object with a hole which the boundaries of containers are equal and boundaries of the holes too, but the hole is placed in other position of the container and then they are not the same object). The orientation is fixed using Frank's Cardinal Reference System (CRF) described before. The CRF is defined by placing its origin into the centroid calculated with the definition of the centroid of a close non regular polygon given in [15]. [15] calculates the centroid ($\alpha_{1,0}$ is the x coordinate and $\alpha_{0,1}$ is the y coordinate) in basis of the area (α) as:

$$\alpha = \frac{1}{2}\sum_{i=1}^{n} x_{i-1}y_i - x_i y_{i-1}$$

$$\alpha_{1,0} = \frac{1}{6\alpha}\sum_{i=1}^{n}(x_{i-1}y_i - x_i y_{i-1})(x_{i-1}+x_j)$$

$$\alpha_{0,1} = \frac{1}{6\alpha}\sum_{i=1}^{n}(x_{i-1}y_i - x_i y_{i-1})(y_{i-1}+y_j)$$

We call centre (C) to the orientation occurred when the hole is placed environ the centroid, then all the orientations hold.

Then once the CRF is placed in the object the orientation of the hole with respect to the object is calculated, for instance figure 6 calculates the orientation of the hole with respect to the container, obtaining that the hole is [NE,E,SE] oriented inside the container.

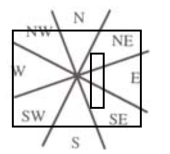
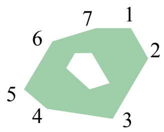


Figure 6. Example of the Orientation Calculation of a hole with respect to its container.

When several orientations hold for a given hole, then the orientation is fixed to a set of all the orientations that holds between the hole and the container, as figure 6 shows.

3 THE COMPLETE DESCRIPTION OF A SHAPE

Given a polygonal shape its complete description is defined as a set as [type,[Colour, $[A_1,C_1,L_1]\ldots[A_n,C_n,L_n]]$,($CI_i$, Orientation, $[[A_{H1},C_{H1},L_{H1}]\ldots[A_{Hj},C_{Hj},L_{Hj}]])^m$], where n is the number of vertices of the container, j is the number of vertices of the holes and m is the number of holes. The type belongs to the set [without-holes, with-holes], Colour is the RGB colour of the piece described by a triple as the set [R,G,B] for the Red, Green and Blue coordinates, $A_1,\ldots,A_n$, $C_1,\ldots,C_n$ and $L_1,\ldots,L_n$ are the qualitative angle, convexity type, and relative length of the vertices of the container respectively. $A_{H1},..A_{Hj}$, $C_{H1},\ldots,C_{Hj}$ and $L_{H1},\ldots L_{Hj}$, are the qualitative angle, convexity type, and relative length of the vertices of the hole. The string CI_i, Orientation, $[[A_{H1},C_{H1},L_{H1}]\ldots[A_{Hj},C_{Hj},L_{Hj}]]$ is repeated for each hole inside the container. Orientation is one of the orientation relations given by the CRF. Therefore, in order to describe completely a shape first we have to repeat the process described in section 2.3 for describe the boundary of the container and the boundary of the holes, beginning by the vertex numbered by 1, until the last vertex is characterised. Then the colour is stored as RGB coordinates, the orientation relation between the container and the hole is calculated using the CRF and the final set (string) with the characteristics of the shape is constructed.



QualShape(S)=[with-holes,[[0,0,0], [obtuse,convex,bigger], [obtuse,convex,smaller], [obtuse,convex,smaller], [obtuse,convex,bigger], [obtuse,convex,smaller], [obtuse,convex,smaller], [obtuse,convex,bigger]], CIi, C, [[acute,convex,smaller], [obtuse,convex,bigger], [obtuse,convex,smaller], [obtuse,convex,smaller], [obtuse,convex bigger]]].

Figure 7. Example of a green (RGB = 159R,207G,169B) shape with a hole.

4 THE MATCHING PROCESS

The matching process is made as follows, first the qualitative description of the object taken as reference is constructed as defined in previous sections, and then the qualitative description of the other object to match is constructed up to the description of the container, it means that the holes are not yet described. With this two strings we compare if both are of the same type (with or without holes), same colour and the containers are equal. For comparing the colour qualitatively the Delta E distance between colours is used. The Delta E distance using RGB colour systems is calculated as:

Given two colours in RGB, named C1 determined by (R1,G1,B1) and C2 determined by (R2,G2,B2), then the Delta E distance between colours is calculated as the Euclidian distance between the RGB coordinates of each colour as:

$Delta\_E(C1,C2)=((R1-R2)^2+(G1-G2)^2+(B1-B2)^2)^{1/2}$

If the Delta_E is less than 0,2 it is because an experimented human eye in the recognising of colours field cannot differentiate between the two colours.

To compare the containers the algorithm *ComparingVertices* is applied, which is a cyclical ordering matching algorithm which given two set of vertices, returns if both strings are equal cyclically and the vertex in the second object which corresponds to the vertex number 0 in the first one. If both sets are not equal the vertex in the second set is not found, therefore a –1 value is assigned.

Algorithm ComparingVertices (INPUTS: SetVertices1, SetVertices2, OUTPUTS: vertex02, equal){

```
N=Calculus size SetVertices1
M=Calculus size SetVertices2
If N == M then {
        //Both sets have the same number of vertices
        For (I=0;I<N-1;I++) {
           For (J=0;J<N-1;J++){ //cyclic comparison
        //Compare Vertex1(0) of SetVertices1 with Vertex2(j) of
        //setVertices2
              If Vertex1(0) == Vertex2(j) {
                    Num=0 //Init a counter
                    For (K=1;K<N-1;K++){
                      If (Vertex1(K)==Vertex2(J+1%N)) then {
                         NUM++;}
                    If (NUM==N){
                       Return equal=true;
                    Return vertex02=j;
                       Break }
                 } //For K
              } // If Vertex1(0) ==Vertex2(j)
           } //For J.
        } //For I
        If (NUM<>N) {
           Return equal=false;
          Return vertex02= -1;}
 } //If N==M
else {
 Return false;}
} //End Algorithm
```

If the objects have not holes the process finishes here.

This way to start the matching process is motivated due to the objects with holes that are found rotated with respect to the reference object to compare will describe the holes in other orientation to the one given to the reference object being both the same object. Then once we obtain that both objects are equal up to the container, and both contains holes, the string describing the holes of the second object (not the reference object already completely described) is constructed by following next steps:

1. Each hole in the object is numbered as being the vertex number 1 the one closest to the vertex which corresponds to the vertex 0 in the reference object, calculated when the cyclic comparison has been made.
2. Include the string CIi in the qualitative description of the object for the first hole.
3. Calculate the orientation of the first hole with respect to the container placing the NW of the RS oriented to the vertex which corresponds to the vertex 0 in the reference object, and include it in the qualitative description of the object.
4. Calculate the qualitative description of the boundary of the first hole ($[[A_{H1},C_{H1},L_{H1}]\ldots[A_{Hj},C_{Hj},L_{Hj}]]$) as it has been done for the container. Include this description in the qualitative description of the object.
5. Repeat steps 2,3, and 4 for each hole inside the object.

Once the qualitative description of the second object is completed, then first we compare the number of holes, if both objects have the same number of holes we continue comparing, and we compare each hole of the reference object with the holes of the other object by doing a non cyclic comparison, in order to allow that cases as figure 8 are considered as not equal as it is the case, because following a cyclic comparison for the holes they will be classified as equals. If all the holes in the reference object have a matching hole in the second object both objects are equal. The algorithm returns false when same of the features of both objects are not equal and it does not follows with the comparison.

Figure 8. Two different objects with identical holes in different positions.

5 APPLICATIONS

The theory here presented has been already applied inside an application which final main objective is the automatic and intelligent assembling of mosaic borders using robot arms in the ceramic industry. A mosaic border is made from different tiles of different shapes, colours and sizes that once they are assembled they create a unique border with high added value (figure 9).

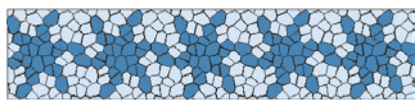
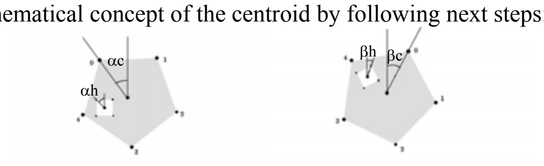


Figure 9. Example of a mosaic border design.

The theory has been implemented such that given as entry an image with different tiles (from now we call it as image) and a vectorial image of the design of the mosaic border (design), the application has to recognise which tile in the image appears in the design and match it against one representation of the tile in the design, therefore the design will contain all the reference objects. Moreover, as later, the application has to allow a robot arm to place the tile in its correct place and orientation, and the tile can appear in a different orientation in the image and the design, then the angle of rotation to place the tile in the correct orientation according to the design is calculated. The angle of rotation δ is calculated using the mathematical concept of the centroid by following next steps:



a) b)

Figure 10. Example of a mosaic border design

1. Find the vertex (vertex I) of the object in the design which corresponds to the upper-most left vertex of the tile in the image (vertex 0). If the object has a hole it is necessary to find the vertex I of the boundary of the container designed (vertex IC) with respect to the vertex 0 of the boundary of the container in the image, and the vertex I of the boundary of the hole (vertex IH) in the design with respect to the vertex 0 of the hole in the image.
2. Calculus of the angle α between the straight line following the direction vector along the x axes and crossing the centroid and the straight line crossing the vertex 0 of the tile and the centroid. If the object has holes it is necessary to calculate this angle α (called αc) for the container, and the angle α for the hole using the centroid of one of the holes (αh) and the vertex 0 of the hole selected (figure 10a).
3. Calculus of the same angles in the object in the design as it has be done in step 2, called βc (container) and βh (hole in the design corresponding to the one selected in 2) (figure 10b).
4. Calculus of the angle of rotation δ as:

If the object has not a hole then:
if $(\beta\text{-}\alpha)>0$ *then* $\delta=(\beta\text{-}\alpha)$ *else* $\delta=(360+(\beta\text{-}\alpha))$
Else
If $(\beta h\text{-}\alpha h)>(\beta c\text{-}\alpha c)$ *then //The angle is determined by the holes*
if $(\beta h\text{-}\alpha h)>0$ *then* $\delta=(\beta h\text{-}\alpha h)$ *else* $\delta=(360+(\beta h\text{-}\alpha h))$
Else //The angle is determined by the containers
if $(\beta c\text{-}\alpha c)>0$ *then* $\delta=(\beta c\text{-}\alpha c)$ *else* $\delta=(360+(\beta c\text{-}\alpha c))$

Thus, the tile in the image has to be rotated δ from its centroid. Moreover, as in this application the size of the objects is an important feature (for instance two squares of very different size are not the same piece), then the area of the shapes is considered. The area is needed too for the calculus of the centroid of the shapes, therefore we do not add more computational cost. The area once more is compared in a qualitative way. The limit to determine two tiles as the same is given by the joint (space leaved between two tiles when they are assembled). As the joint differs from one design to other it is given by the user of the application.

6 CONCLUSIONS

The Qualitative Theory of Shape defined in this article allows us to reason about shape in a qualitative way as human beings do. Moreover, most of the qualitative approaches developed nowadays are used for reasoning about object position, and the theory presented here allows us to use the same method to reason about position and shape. The theory proposed is a example of ordering information approach for shape description which interest relies in the fact that it is less constrained than metrical information but more constrained than topological information, which will not allow us to determine the convexity or concavity of the shape, neither the length of edges, nor the angle types.

The proposed theory has been applied to the recognition of tiles in a mosaic border design in order to allow an automatic and intelligent assembling of mosaic borders in the ceramic industry. A qualitative theory for this application presents several advantages against the use of a quantitative theory, as the managing of uncertainty. Moreover, this application could be used to the recognition of objects of any other vectorial design.

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