From belief change to obligation change in the Situation Calculus.

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Abstract.
A solution to the frame problem in the context of belief change has been defined in the framework of Situation Calculus. In this paper we show how this solution can be adapted to obligation change. For that purpose ideality levels are assigned to situations in the same way as plausibility levels are assigned to situations in the context of belief change. However, it is shown that there are deep differences between the evolution of beliefs and the evolution of obligations.

1 Introduction

There are many application domains where obligations may change according to a regulation when some particular actions are performed.

In this paper we analyse and propose a formalism to represent obligation change when actions are performed. There are many works in the literature about a similar problem so called "defeasible obligations" [6, 8, 5]. The big issue is to solve the "frame problem". Our solution is based on an adaptation of its solution proposed in the framework of Situation Calculus [9, 10, 4, 11, 7].

2 A brief introduction to belief change in the Situation Calculus

Situation Calculus is a typed first order logic where fluents have exactly one argument of the type situation. For example, the meaning of $alt(x, s)$ is that in the situation s we are at the altitude x. Situations are terms of the form s or $do(a, s)$, where $do(a, s)$ represents the situation where we are after performance of the action a.

For each fluent p we have to define a successor state axiom of the form:

(S_p) $\forall s \forall a \forall \vec{x}(p(\vec{x}, do(a, s)) \leftrightarrow \Gamma_p^+(\vec{x}, a, s) \vee p(\vec{x}, s) \wedge \neg\Gamma_p^-(\vec{x}, a, s))$

Γ_p^+ and Γ_p^- represent all the conditions that respectively cause p to be true or to be false in $do(a, s)$. For example, Γ_p^+ may be $a = a_1 \wedge q(\vec{x})$ and Γ_p^- may be $(a = a_2 \vee a = a_3) \wedge \neg r(\vec{x})$.

To represent beliefs the designated predicate $K(s', s)$ is introduced. $K(s', s)$ means that s' is compatible with what is believed in s. In addition a plausibility level $pl(s)$ is assigned to each situation s. A proposition ϕ is believed in s, and that is denoted by $Bel(\phi, s)$, if ϕ holds in all the situations accessible by K that have the lowest plausibility level. In formal terms we have:

$$K_{max}(s', s) \stackrel{\text{def}}{=} K(s', s) \wedge \forall s''(K(s'', s) \rightarrow pl(s') \leq pl(s''))$$

$$Bel(\phi, s) \stackrel{\text{def}}{=} \forall s'(K_{max}(s', s) \rightarrow \phi[s'])$$

To formalise belief change we have to characterise the new set of accessible situations and the new truth values of the fluents after performance of an action. The new truth values are defined by the successor state axioms in each situation. The accessible situations are defined depending on the type of performed action. Two types of actions are considered: observations, of the form α_i, that allow to know the truth value of some given proposition ϕ_i, and do not change the physical state of the world, and other actions that do change the physical state of the world and change the set of beliefs accordingly.

The set of accessible situations after performance of an action is defined by the axiom (S_K), where $\alpha_1, \ldots, \alpha_n$ is the set of **all** the observation actions, and $\phi_1, \ldots, \phi_n$ is the set of corresponding sentences whose truth values are obtained from the observation actions.

(S_K) $\forall s \forall s'' \forall a(K(s'', do(a, s)) \leftrightarrow \exists s'(K_i(s', s) \wedge s'' = do(a, s') \wedge ($
$(\neg(a = \alpha_1) \wedge \ldots \wedge \neg(a = \alpha_n))$
$\vee\, a = \alpha_1 \wedge (\phi_1(s) \leftrightarrow \phi_1(s'))$
$\ldots$
$\vee\, a = \alpha_n \wedge (\phi_n(s) \leftrightarrow \phi_n(s'))))$

This axiom says that s'' is accessible from $do(a, s)$ iff it is the successor of a situation accessible from s (condition $\exists s'(K_i(s', s) \wedge s'' = do(a, s'))$) and one of the following conditions holds. Either the action a is not an observation action (condition $\neg(a = \alpha_1) \wedge \ldots \wedge \neg(a = \alpha_n)$) or it is some observation action α_i and the truth value of ϕ_i is the same in s and s' (condition $a = \alpha_i \wedge (\phi_i(s) \leftrightarrow \phi_i(s'))$).

3 Obligation change

Now we analyse how the above formalisation of belief change can be adapted to formalise obligation change.

We have considered that it is not an over simplification to adopt the Standard Deontic Logic (SDL) which is a logic of type (KD) like the logic of beliefs which has been presented in the previous section.

We introduce an accessibility relation $O(s', s)$ whose meaning is that what is the case in the situation s' is ideal for the situation s, and we have assigned an ideality level $id(s)$ to each situation s. A proposition ϕ is obligatory in s, and that is denoted by $Obg(\phi, s)$, if ϕ holds in all the situations that have the lowest ideality level. In formal terms we have:

$$O_{max}(s', s) \stackrel{\text{def}}{=} O(s', s) \wedge \forall s''(O(s'', s) \rightarrow id(s') \leq id(s''))$$

$$Obg(\phi, s) \stackrel{\text{def}}{=} \forall s'(O_{max}(s', s) \rightarrow \phi[s'])$$

Moreover, permission and forbiddence are defined from obligation as usual.

$$Perm(\phi, s) \stackrel{\text{def}}{=} \neg Obg(\neg\phi, s)$$

$$Forb(\phi, s) \stackrel{\text{def}}{=} Obg(\neg\phi, s)$$

However, obligations do not change in the same way as beliefs do. The deep reason of this difference is that ideal situations do not change "according to" changes in the real situations like for beliefs.

We have to distinguish three different types of actions:

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1. actions which do not change obligations,
2. physical actions which have the effect to change obligations, and
3. non physical actions which have the effect to change obligations.

An example of action of type 1 is to walk in a street. An example of action of type 2 is to enter in a public building, which has the effect that it is forbidden to smoke. An example of action of type 3 is to give the permission to someone to smoke at home.

An action of type 1 changes beliefs accrodingly in the sense that beliefs about the position change as long as we walk, and that is formally represented by applying the successor state axioms to the situations accessible by K. However, since an action of type 1 does not change the obligations the set of situations accessible by O remains unchanged after performance of this action.

The second significant difference between belief change and obligation change is that we have to distinguish actions that create new permissions and those that create new obligations. Indeed, in the case of permissions it is sufficient to have one most ideal accessible situation where the permitted property holds, while in the case of an obligation this property must hold in all the most ideal accessible situations. The creation of new obligations are analogous to observation actions, but there is no operation analogous to the creation of a permission in terms of beliefs.

If we denote by $perm.p$ (respectively by $obg.p$) the action the effect of which is that p is permitted (respectively p is obligatory), we can easily check that these two kinds of actions are enough to cover all the kinds of changes. Indeed, $\neg Perm(p, do(a,s))$ is caused by $a = obg.\neg p$, and $\neg Obg(p, do(a,s))$ is caused by $a = perm.\neg p$.

The situations accessible by O after an action $perm.p$ that causes a permission is defined in general by:

$\forall s \forall a(a = perm.p \rightarrow$
$(O(s', do(a,s)) \leftrightarrow O(s',s) \wedge (p(s') \vee \exists s''(Os'', s') \wedge id(s'') \leq id(s') \wedge p(s'')))))$

The situations accessible by O after an action $obg.p$ that causes an obligation is defined in general by:

$\forall s \forall a(a = obg.p \rightarrow$
$(O(s', do(a,s)) \leftrightarrow O(s',s) \wedge (p(s') \vee \exists s''(id(s'') < id(s') \wedge p(s'')))))$

Notice that for permissions we have the condition $id(s'') \leq id(s')$ while for obligations we have the condition $id(s'') < id(s')$.

The axiom (S_O) defines what is the new set of ideal accessible situation after performance of any kind of action.

Let us call $\alpha_{i,1}, \ldots, \alpha_{i,m_i}$ (respectively $\beta_{i,1}, \ldots, \beta_{i,p_i}$) the set of **all** the actions that cause that it is permitted (respectively obligatory) that p_i. We adopt the notation:

$normative(a) \stackrel{\text{def}}{=} a = \alpha_{1,1} \vee \ldots \vee a = \alpha_{n,m} \vee a = \beta_{1,1} \vee \ldots \vee a = \beta_{n,p} \vee a = perm.p_1 \vee \ldots \vee a = perm.p_n \vee a = obg.p_1 \vee \ldots \vee a = obg.p_n$

Intuitively $normative(a)$ characterises **all** the actions that have normative effects.

The axiom schema (S_O) says that the situation s' is accessible from the situation $do(a,s)$ iff it was accessible from s and one of the following conditions holds. Either a is not a normative action or it is a normative action that cause that some p_i is permitted (respectively obligatory) and the condition: $p(s') \vee \exists s''(id(s'') \leq id(s') \wedge p(s''))$ (respectively $p(s') \vee \exists s''(id(s'') < id(s') \wedge p(s''))$) holds.

(S_O) $\forall s \forall s' \forall a(O(s', do(a,s)) \leftrightarrow$
$O(s',s) \wedge (\neg normative(a)$
$\vee(a = \alpha_{1,1} \vee \ldots \vee a = \alpha_{1,m_1} \vee a = perm.p_1) \wedge (p_1(s') \vee$
$\exists s'_1((id(s'_1) \leq id(s')) \wedge p_1(s'_1)))$
$\vee(a = \beta_{1,1} \vee \ldots \vee a = \beta_{1,p_1} \vee a = obg.p_1) \wedge (p_1(s') \vee$
$\exists s'_1((id(s'_1) < id(s')) \wedge p_1(s'_1))))$
$\ldots$
$\vee(a = \alpha_{n,1} \vee \ldots \vee a = \alpha_{n,m} \vee a = perm.p_n) \wedge (p_n(s') \vee$
$\exists s'_1((id(s'_1) \leq id(s')) \wedge p_n(s'_1)))$
$\vee(a = \beta_{n,1} \vee \ldots \vee a = \beta_{n,p} \vee a = obg.p_n) \wedge (p_n(s') \vee$
$\exists s'_1((id(s'_1) < id(s')) \wedge p_n(s'_1))))))$

4 Conclusion

A model of evolution of obligations has been proposed in the framework of Situation Calculus with the axiom schema S_O. We have selected the new set of ideal situation in order to minimise obligation change.

Also, it has been shown that belief change cannot be trivially adapted to obligations.

It is also worth noting that the final result is that we have a formalisation in Situation Calculus of both belief change and obligation change. Since there is no restriction about the sentences in the scope of the "belief operator" or of the "obligation operator" we can represent nested operators, like beliefs about obligations or obligations about beliefs. That leaves the door open for many practical applications.

The analysis of obligations about evolution of the world deserves further investigations. We believe it can be accomodated in the same framework.

Another approach to the formalisation of obligation change to be investigated might be to restrict obligations to obligations about sentences that can be represented by literals, and to define successor state axioms for obligations as we did in [3] for beliefs.

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