Miss Scarlett in the Ballroom with the Lead Piping

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Abstract. Temporal logics of knowledge are useful for reasoning about situations where the knowledge of an agent or component is important, and where change in this knowledge may occur over time. Here we use temporal logics of knowledge to reason about the game Cluedo. We show how to specify Cluedo using temporal logics of knowledge and prove statements about the knowledge of the players using a clausal resolution calculus for this logic.

1 INTRODUCTION

Temporal logics of knowledge [4] are useful for specifying dynamic systems that change over time which also involve informational aspects relating to the knowledge of agents. Cluedo, commercially produced by Hasbro [5], is a board game that involves reasoning about the knowledge of other players' cards. We show how the game can be specified using a temporal logic of knowledge and how moves in the game correspond to additional knowledge for one or more of the players. Using a simplified version of the game, we show how to prove certain inferences using a resolution-based approach. The contribution of the paper is a case study using temporal logics of knowledge to represent and reason about the game Cluedo. In particular we demonstrate the suitability of this logic for specifying Cluedo and show how to verify the derived knowledge of players using a resolution calculus for this logic.

The logic, KL_n, we consider is the fusion of linear time temporal logic with finite past and infinite future combined with the multi-modal logic S5. We allow the usual set of temporal operators including "$\bigcirc$", in the next moment in time, and use the modal operator "K_i" to denote the knowledge of agent i. For the syntax and semantics of KL_n see for example [2, 3]. We note that KL_n does not include an operator for common knowledge. To verify properties of the specification we carry out proofs by refutation using resolution for temporal logics of knowledge [2, 3].

2 THE GAME CLUEDO

Cluedo is a board game where players gather information about a murder. The suspects, murder weapons and room where the murder took place are represented by playing cards. One from each of these sets is removed and placed, without any of the players seeing, in an envelope to represent the actual murderer, murder weapon and location of the murder. The remaining cards are shuffled and dealt out to the players. Players take it in turns to make *suggestions*, a triple: suspect, weapon and room. If the player to their left has one of these cards it is shown secretly to the player making the suggestion. The other players can see a card has been shown to the player making the suggestion but do not know its identity. If the player to the left does not hold one of the three cards in the suggestion, she publicly declares this and the player to her left must try and show the suggesting player one of the cards. This continues until a card has been shown to the suggesting player or no card has been shown by any player for this suggestion. Players use the knowledge about the cards in their hand and knowledge about cards other players may or may not hold to eliminate suspects, weapons and rooms from their enquiries. When a player knows the murderer, murder weapon, and room she makes an *accusation* and checks the hidden murder cards. If she is correct she wins the game. Note, whilst each player makes many suggestions during a game, at most one accusation is made by each player during a game so the player should be certain about the murderer, weapon and location before making an accusation.

The commercial version of the game is produced by Hasbro [5] and involves a board which represents the rooms in the house and access between them. As we are interested in representing and reasoning about knowledge we ignore this aspect of the game. In the full game there are six suspects, six weapons and nine rooms.

3 SPECIFYING A CLUEDO GAME

First we will reduce the game to illustrate how we may specify actions in the game. The changes in knowledge relating to moves in the game is Cluedo has been described in [7] but using a different logic. Let us assume that we have simply four suspects (Prof. Plum, Rev. Green, Col. Mustard and Miss Scarlett), four weapons (lead piping, spanner, revolver and rope) and no rooms. It is easy to scale this up to the full number of suspects, weapons and rooms. In our example we assume three players Catherine, Wendy and Jane.

Let the set of players $Ag = \{c, w, j\}$. First we use propositions to show who holds each of the cards, where $i \in \{c, w, j\}$.

- r_i is true if player i holds Miss Scarlett
- g_i is true if player i holds Rev Green
- y_i is true if player i holds Col. Mustard
- b_i is true if player i holds Prof. Plum
- l_i is true if player i holds the lead piping
- s_i is true if player i holds the spanner
- v_i is true if player i holds the revolver
- p_i is true if player i holds the rope

A suspect is denoted as the murderer, or a weapon as the murder weapon by having m as a suffix, i.e. r_m is true if Miss Scarlett is the murderer and similarly for the other suspects and weapons above.

At The Start of the Game Initially (and throughout the game) one of the suspects must be the murderer $(r_m \lor g_m \lor y_m \lor b_m)$ and one of the weapons is the murder weapon $(l_m \lor s_m \lor v_m \lor p_m)$. Initially (and throughout the game) each card must be held by one of the players or it must be the murderer or murder weapon. For example for Miss

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Scarlett $r_c \vee r_w \vee r_j \vee r_m$ and similarly for the other suspects and weapons.

If a player (eg Catherine) holds a card (eg Miss Scarlett) then the other players don't hold it and it can't be the murderer or murder weapon i.e. $r_c \Rightarrow (\neg r_w \wedge \neg r_j \wedge \neg r_m)$ (and similarly for Wendy and Jane). If a card is the murder suspect or weapon then none of the players can hold that card, i.e. $r_m \Rightarrow (\neg r_w \wedge \neg r_j \wedge \neg r_c)$. Similar axioms hold for the other weapons and suspects.

All of the axioms in this section are known by each player and the other players know this etc (i.e. they are common knowledge).

After the Deal After the deal, each player knows they hold the cards that they have been dealt and knows that they don't hold the cards they haven't been dealt. For example if Catherine is dealt Miss Scarlett and Rev. Green then $K_c r_c \wedge K_c g_c \wedge K_c \neg y_c \wedge K_c \neg b_c \wedge K_c \neg l_c \wedge K_c \neg s_c \wedge K_c \neg v_c \wedge K_c \neg p_c$

After a Suggestion If Catherine makes the suggestion Miss Scarlett and the lead piping, then there are two options, either the next player has one of these cards and shows her or the next player does not have one of these cards. Let us assume that Wendy first tries to answer the suggestion.

For the former, i.e. Wendy does not hold Miss Scarlett or the lead piping, Wendy states that she does not hold them for all to hear. From this statement $\neg r_w$ and $\neg l_w$ becomes known to everyone (and is common knowledge) i.e. $K_i \neg r_w \wedge K_i \neg l_w$ for $i \in \{c, w, j\}$. For the latter i.e. Wendy holds one of Miss Scarlett or the lead piping, several inferences can be made. Firstly that Wendy holds Miss Scarlett or the lead piping is known to each player i.e. $K_i(r_w \vee l_w)$ for $i \in \{c, w, j\}$ and this again is common knowledge. Also Catherine learns the identity of one of the cards Wendy holds (say it is the lead piping) i.e. $K_c l_w$. Further, Wendy knows Catherine knows this etc.

Lastly if a player (say Catherine) makes a suggestion (for example Miss Scarlett and the lead piping), all the other players state they do not hold one of these cards and the suggesting player, Catherine, does not make an accusation then each player knows that Catherine must hold either Miss Scarlett or the lead piping, i.e. $i \in \{c, w, j\}$ $K_i(r_c \vee l_c)$. This is again common knowledge.

The End of the Game The game ends when one of the players know the murderer and the murder weapon i.e. $K_i(x_m \wedge z_m)$ where $i \in \{c, w, j\}$ and x is one of r, g, y, b and z is one of l, s, v, p.

Dealing with Time We assume that each element of the game where the knowledge changes occurs at the next time point. For example we assume that the deal occurs at time one, the first time a suggestion is answered is at time two etc.

If a particular player knows who holds (or does not hold) a card then we assume they do not forget this information, i.e.

$$K_i x_k \Rightarrow \bigcirc K_i x_k \text{ or } K_i \neg x_k \Rightarrow \bigcirc K_i \neg x_k$$

where $i, k \in \{c, w, j\}$ and $x \in \{r, g, y, b, l, s, v, p\}$. That is *if a player knows that someone (doesn't) holds (hold) a card then in the next moment they know that person (doesn't) hold that card*, i.e. players don't forget knowledge relating to holding cards.

Common Knowledge As noted in Section 1 the KL_n logic does not include operators for common knowledge. Statements for example "Wendy answers she doesn't hold Miss Scarlett or the lead piping" mean that the fact that Wendy doesn't hold Miss Scarlett or the lead piping become common knowledge at some point during the game. As there is no explicit common knowledge operator we must explicitly include (one or more) knowledge operators around the statement. That is we must explicitly state the depth of modal operators we require.

4 VERIFICATION USING CLAUSAL RESOLUTION

Having specified a Cluedo game proofs are carried out using a resolution calculus for KL_n. For the full details of the resolution method see [2, 3]. The specification for a particular Cluedo game can be found in [1]. Resolution proofs to show the knowledge of a player at some point in the game are also given.

5 CONCLUSIONS AND RELATED WORK

We have used a propositional linear-time temporal logic of knowledge to specify and verify properties of the game Cluedo. Full details are provided in [1].

Combinations of modal and temporal logics have been used to specify complex situations. For example BDI logics [6], the fusion of branching-time (CTL or CTL*) temporal logics with the modal logics KD45 for belief, and KD for desire and intention, are used to specify properties multi-agent systems. Tableau based proof methods are also given for these logics. The muddy children problem is a well known problem relating to reasoning about knowledge. The problem is specified using epistemic logics in [4] and is specified and verified using temporal logics of knowledge in [3].

The specification of Cluedo game actions has been carried out in [7] in a dynamic epistemic logic (the combination of dynamic logic and a logic of knowledge allowing common knowledge). Unsurprisingly the knowledge gained from moves in the game is the same as described in this paper except the common knowledge resulting from some moves can be explicitly stated. The focus of [7] is the specification of the knowledge actions rather than verification. The paper has no axiomatisation for the logic and decidability is not discussed.

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