Context Dependence in Multiagent Resource Allocation

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Abstract. A standard assumption in studies of multiagent resource allocation problems is that the value an individual agent places on its assignment remains unchanged by any redistribution of the remaining resources among the other agents. This assumption renders impossible analyses of scenarios where the utility an agent attaches to a particular set of resources is determined by factors other than the resource set itself. Thus an agent's perception of what its allocation is worth may be tempered by its view of what other agents in the system may own, e.g. if working within a coalition a particular allocation may assume a greater value if other coalition members hold certain resources. In this paper we develop a model for examining such *context dependent* valuations and consider various decision problems related to the existence of context dependent allocations satisfying various criteria.

1 Introduction

Mechanisms for reasoning about allocations of resources within a group of agents form an important body of work within the study of multiagent systems. Typical abstract models derive from game-theoretic perspectives in economics and among the issues that have been addressed are strategies that agents may use to negotiate, e.g. [8, 10, 11], and protocols for negotiation in agent societies, e.g. [2, 4, 5, 6, 9]. A formal definition of the standard resource allocation setting is given in Section 2 below, however, the analyses of this paper arise from one particular aspect of this model. An implicit assumption it makes is that the value an agent, A_i, places upon a subset, S, of some set of resources $\mathcal{R}$, is *context independent*. In other words, this value, $u_i(S)$, does not vary regardless of what allocation of the resources $\mathcal{R} \setminus S$ is used for the other agents in the systems. It is not difficult, however, to envisage situations which such context independent interpretations of utility have difficulty modelling. Thus, in a 3 agent system, A_1 and A_2 may wish to act in partnership against A_3 in determining a partition of a resource set. In this context the value that A_1 places upon a particular subset S may vary according to which subset of the remaining resources A_2 obtains. In addition to such coalitional settings, one may wish to model situations whereby an individual agent will assess a given assignment as having greater worth if it arises in a context for which some other agent is not granted certain resources. As a more concrete example of where context dependent evaluation is significant one can consider partnership games such as Bridge, in which setting it is well-known that the 'value', in terms of trick taking potential, of a given hand may depend significantly on the distribution of the remaining cards among the other three players. In this paper we develop an approach to the analysis of context dependent resource allocation settings, the central component of which allows agents to discriminate among different overall allocations under which it receives a particular set of resources. The basic approach is given with other definitions in Section 2. The main aim of this paper is to initiate the study of our context dependent model by considering a number of 'natural' decision questions within it. A selection of these together with their complexity classification is presented in Section 3. We refer the reader to [7] for more a more detailed exposition and proofs. Conclusions are given in the final section.

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2 Definitions

The basic setting we are concerned with is encapsulated in the following definition.

Definition 1 *A* resource allocation setting *is defined by a triple $\langle \mathcal{A}_n, \mathcal{R}_m, \mathcal{U} \rangle$ with $\mathcal{A}_n$ a set of agents and $\mathcal{R}_m$ a collection of (non-shareable) resources. A* utility function, *u, maps subsets of $\mathcal{R}_m$ to rational values. Each agent $A_i \in \mathcal{A}$ has associated with it a particular utility function u_i, so that $\mathcal{U}$ describes these. An* allocation *P of $\mathcal{R}_m$ among $\mathcal{A}_n$ is a partition of $\mathcal{R}_m$. We use the notation $\Pi_{n,m}$ to indicate the set of all distinct allocations of $\mathcal{R}_m$ among $\mathcal{A}_n$, noting that there are exactly n^m of these. The value $u_i(P_i)$ is called the* utility *of the resources assigned to A_i.*

The main aspect of the form of Definition 1 that we wish to address concerns its assumption that for any $S \subseteq \mathcal{R}_m$ and agent A_i, in allocations P and Q under which A_i receives S, the value $u_i(S)$ within P is exactly the same as its value within Q, i.e. $u_i(S)$ is invariant over all allocations of $\mathcal{R}_m \setminus S$ among the other agents.

The basic mechanism we use to allow an agent A_i to discriminate between such allocations is that of a *ranking function*.

Definition 2 *A* prioritised resource allocation setting *(PRAS) is defined by a pair $\langle \langle \mathcal{A}_n, \mathcal{R}_m, \mathcal{U} \rangle, \mathcal{V} \rangle$ where $\mathcal{V}$ defines a collection of n* ranking functions. *The ranking function for A_i, ρ_i, maps each $P \in \Pi_{n,m}$ to a non-negative integer $\rho_i(P)$ in the range $[0, n^m - 1]$. For a given allocation, $P \in \Pi_{n,m}$, the n-tuple of values $\langle \rho_1(P), \rho_2(P), \ldots, \rho_n(P) \rangle$ is called the* preference profile *of P. We say that an n-tuple, $\langle k_1, k_2, \ldots, k_n \rangle$ of non-negative integer values is an* attainable profile *if there is an allocation, P, such that $\rho_i(P) \leq k_i$ for each $1 \leq i \leq n$. If P and Q are allocations under which $\rho_i(P) < \rho_i(Q)$ we say that A_i* prefers the allocation *P* to the allocation *Q.*

The concept of rank function provides one mechanism for an agent to discriminate between the $(n-1)^{m-|P_i|}$ distinct allocations to $\mathcal{A}_n \setminus \{A_i\}$ that are consistent with A_i being assigned $P_i \subseteq \mathcal{R}_m$. In addition we obtain an approach that can be used to describe a number of ideas examined in earlier work. Consider, for example, the concept of an allocation being "envy-free" where each agent values what it has been given at least as highly as it would value the resources granted to any other. This concept is easily encapsulated within a prioritised resource allocation setting: define the ranking function $\rho_i(P)$ as $\rho_i(P) = |\{ j : u_i(P_j) > u_i(P_i)\}|$, so that an allocation is envy-free if and only if $\rho_i(P) = 0$ for each i.

The formulation of attainable profile requires only that each agent views an allocation to be *at least* as good as the preference rank indicated. Within any PRAS, there will, for each agent, be some set of allocations that it regards as most preferred. One area of interest concerns allocations that achieve the maximal preferred status with respect to arbitrary subsets (or *coalitions*) from the set of all agents. Thus, using μ_i to denote the minimal attainable value of ρ_i, we can classify P as *optimal* (for A_i), if $\rho_i(P) = \mu_i$; *ideal* if $\rho_i(P) = 0$, extending these to *subsets*, $\mathcal{C}$, via the concept of a *consensus*: one that is optimal for each agent in $\mathcal{C}$. We can, additionally, introduce a notion of allocations considered with respect to *opposing coalitions* – $\mathcal{C}$, $\mathcal{D}$ – so that $\mathcal{C}$ *can obstruct* the coalition $\mathcal{D}$ if there is some *obstructive set* of resources which can be distributed among the members of $\mathcal{C}$ to form a consensus for C but with every allocation of the remaining resources among $\mathcal{D}$ *failing* to be optimal for any member of $\mathcal{D}$. One consequence arising from this idea is that particular sets, S say, acquire significance in terms of the current allocation and given coalitions, $\mathcal{C}$ and $\mathcal{D}$: they may be *critical* in the sense that were $\mathcal{C}$ to acquire S then it would be able to obstruct $\mathcal{D}$. Thus in such cases, it would be in the interests of $\mathcal{C}$ to acquire the missing elements of S while, similarly, agents in $\mathcal{D}$ would not only seek to prevent this, but would also have to recognise their potential occurrence.

We note that we may recover the standard mechanism for an agent to distinguish between allocations (within a non-prioritised setting) merely by considering a decreasing order of the 2^m potential values $u_i(S)$ where $S \subseteq \mathcal{R}_m$ and fixing $\rho_i(P)$ to be the position of $u_i(P_i)$ within this, so that higher valued resource subsets are preferred.

3 Decision Problems for Prioritised Settings

Our aim in this preliminary study is to consider prioritised resource allocation settings with respect to complexity issues. We present a number of decision problems that naturally arise in this model.

Definition 3 *The decision problem* Subjective Improvement *(*SI*) takes as an instance a PRAS, an allocation P and an index i with* $1 \leq i \leq n$ *which is accepted if there is an allocation Q with* $\rho_i(Q) < \rho_i(P)$. *The decision problem* Objective Improvement *(*OI*) takes as an instance a PRAS and an allocation P: the instance is accepted if there is an allocation Q for which* $\wedge_{i=1}^{n}(\rho_i(Q) < \rho_i(P))$ *holds.*

In addition we consider the decision problems *Attainable Profile* (AP), *Obstructive Coalition* (OC); and, *Critical Set* (CS), formal definitions of which are easily derived.

Our results are summarised in,

Theorem 1

a. AP, SI, OI, *and the problem of deciding if an* envy-free *allocation is possible, are* NP*–complete (even when* $n = 2$*)*
b. OC *and* CS *are* Σ_2^p*–complete.*

4 Conclusions

The principal contention of this paper is that the oft employed model considered in the study of multiagent resource allocation is insufficiently expressive to address arenas wherein the worth a single agent attributes to its alloted resource is dependent on external factors. We have argued that importing a simple ranking mechanism into the standard setting provides an approach flexible enough to model such context dependent issues, illustrating this view with reference to a select number of natural decision questions whose computational complexity has been classified. These include both problems that encompass related questions in the standard setting, e.g. Subjective Improvement, as well as a number that arise specifically in our prioritised variant, e.g. Obstructive Coalition.

Although we have chosen to present this model from the viewpoint of multiagent resource allocation and evaluation, we note that the issues motivating it are also of great relevance to more general concerns arising from scenarios modelled through some underlying set ($\mathcal{R}$) divided among a finite set of participants ($\mathcal{A}$). Thus if $\mathcal{R}$ is interpreted as a collection of beliefs, attitudes, and facts held by members of $\mathcal{A}$ then we have a framework for considering persuasive argument, e.g. in the scheme of [9], where the force and acceptance of particular claims by one agent depends not only on its own beliefs and attitudes but also on how these relate to the views endorsed by other agents. Since, in principle abstract models of argument and reasoning such as that of Dung [3] could be embedded within a multi-party debate setting, the development of these to describe relative notions of value preferences that has been initiated in the work of Bench-Capon [1] may be defined through our prioritised model.

Finally we note the potentially rich seam of problems that arise in formulating strategies for coalitions to identify consensus allocations, critical sets, and obstructive possibilities. Even a setting comprising only 4 agents yields non-trivial strategic questions for the coalitions involved when these seek either to improve their preference or avoid it degrading.

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