Qualitative Modelling of Planar Robots

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Abstract. This study proposes an approach, the unit circle (UC), to qualitative modelling of a planar robot. A robot is described as a collection of constraints holding among time-varying, interval-valued variables. The UC representation is presented, the continuous motion of the end-effector is evaluated by the change of directions of qualitative angle and qualitative length. Analytical formulas of qualitative velocity and qualitative acceleration are derived. The UC representation of robots concerns a global assessment of system behaviours, it can be used for the purpose of monitoring, diagnosis, and explanation of physical systems.

1 Introduction

Qualitative and quantitative methods are two ways of looking at the world and solving problems. They have their advantages and disadvantages when used as solutions to particular problems. When the tradeoff between computational complexity and accuracy is a major problem, qualitative reasoning methods are usually considered as the preferred solutions. In the area of qualitative analysis of physical systems a number of approaches have been developed. The contributions made to qualitative kinematics can be referred to [1-5].

This paper proposes a novel qualitative modelling scheme for the representation of planar robots, and the approach can be easily extended to spatial robots. This is the first attempt to clearly define the qualitative representation of robots, whose end-effector's position can be described by a qualitative length and a qualitative orientation angle within a unit circle. Qualitative analysis of a robot is constructed in terms of subsets of a unit circle with link sequence constraints. The unit circle (UC) approach also derives qualitative velocity and qualitative acceleration based on qualitative length and orientation angle. Engineering design, like robotic navigation, ultimately normally requires a fully metric description. However, at the early stages of the design process, a reasonable qualitative description would suffice. The field of qualitative kinematics is largely concerned with supporting this type of activity.

2 Qualitative Representation of Planar Robots

A n-link serial robot, combined by links and joints, can be decomposed into n link-based segments, each of which consists of one link and its corresponding joint. Each segment can be described by a qualitative length and a qualitative orientation angle, furthermore the qualitative representation of the end-effector of the robot is provided by the qualitative information of each link segment.

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2.1 Qualitative Position

Definition 1: The qualitative position of the end-effector of a robot can be described by a pair of qualitative position and qualitative orientation in a unit circle, which are provided by combination of qualitative parameters of all segment links.

For a global assessment of a system behaviour, functional rules of qualitative constraints are applied so that the interval-valued parameters of the ith link segment are replaced by the proportion of the interval-valued parameters of the ith link segment to the addition of the length of all link segments. It is noted that the link segments are connected by the link sequence constraints, $l_1, l_2, \cdots, l_i, \cdots, l_n$, based on which, the UC qualitative representation of the position of the end-effector of the n-link robot can be derived.

$$\begin{cases} qp_l = \bigoplus_{i=1}^{n} qp_l^i \,\Big|\, qp_l^i \in UC_{ql} \times \sum_{i=1}^{n} l_i \\ qp_\theta = \bigoplus_{i=1}^{n} qp_\theta^i \,\big|\, qp_\theta^i \in UC_{q\theta_i} \times 2\pi \end{cases} \tag{1}$$

where

$$\begin{cases} UC_{qp_l} = \left[0, \frac{l_{11}}{\sum_{i=1}^{n} l_i}, \frac{l_{12}}{\sum_{i=1}^{n} l_i}, \cdots, \frac{l_{i1}}{\sum_{i=1}^{n} l_i}, \cdots, \frac{l_{n(r_n-1)}}{\sum_{i=1}^{n} l_i}, \frac{\sum_{i=1}^{n} l_i}{\sum_{i=1}^{n} l_i}\right] \\ UC_{qp_\theta^i} = \left[0, \frac{q\theta_{i1}}{2\pi}, \frac{q\theta_{i2}}{2\pi}, \cdots, \frac{q\theta_{i(s_i-1)}}{2\pi}, \frac{2\pi}{2\pi}\right] \end{cases}$$

Here UC_{qp_l} stands for the qualitative length of a unit circle, $UC_{qp_\theta^i}$ for the qualitative orientation angle of the unit circle. Each interval-valued area of the unit circle, $UC_{ql}, UC_{q\theta_i}$ corresponds to the qualitative meaning in domain knowledge representation. The representation conversion of a particular position 'Q' of the end-effector of a n-link serial robot from quantitative to qualitative description is given in Figure 1. The robot is described in terms of Cartesian coordinates, its qualitative representation of the UC is in terms of the qualitative angle and the qualitative length of the end-effector. It is defined by a qualitative vector $\vec{Q'}$ from the origin to the position Q. For example, the qualitative orientation can be divided into $front$, $back$, $left$ and $right$. The qualitative length can be identified by $small$, $equal$ and $larger$; three regions in the qualitative description.

In position mapping from the qualitative representation to continuous quantities, one of standard assumptions in traditional qualitative reasoning is that change is continuous. That is, in addition to qualitative magnitude such as qualitative angles and qualitative lengths in the quantity space, we need to know the direction of change of each variable. Thus for each variable, we describe its qualitative state in terms of its magnitude in the quantity space and its direction of change as follows,

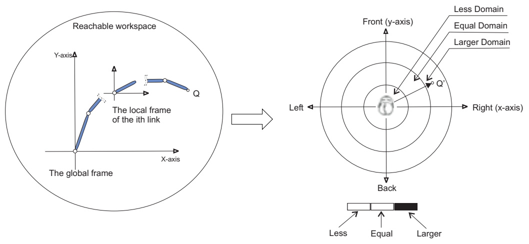

Figure 1. The conversion from quantiative space to qualitative

$$\left[\Delta qp^k\right] = sign\left(qp^{k+1} - qp^k\right) = \begin{cases} + & \Delta qp^k > 0 \\ - & \Delta qp^k < 0 \\ 0 & \Delta qp^k = 0 \end{cases}$$

Where qp_i^k denotes the qualitative position parameters within the kth interval. Then the qualitative position of the end-effector of a robot can be described as follows,

$$[\Delta qp] = \overset{n}{\underset{k=1}{\oplus}} \left[\Delta qp^k\right] = \overset{n}{\underset{k=1}{\oplus}} \left[\Delta qp_l^k\right] \overset{n}{\underset{k=1}{\oplus}} \left[\Delta qp_\theta^k\right]$$

For robotic qualitative kinematics, the continuous motion of robots can be described by the combination of magnitudes of qualitative parameters with their direction of change. The direction of change of a qualitative orientation angle is defined by a qualitative orientation vector, whose direction perpendicular to the corresponding qualitative vector; the direction of change of a qualitative length is defined by a qualitative length vector, whose direction is vertical to the qualitative vector. The anticlockwise direction of qualitative orientation angles is denoted as positive, and the direction of facing the origin of qualitative lengths as positive. We have developed a Matlab-based toolbox UC for robot qualitative modelling, see Figure 2.

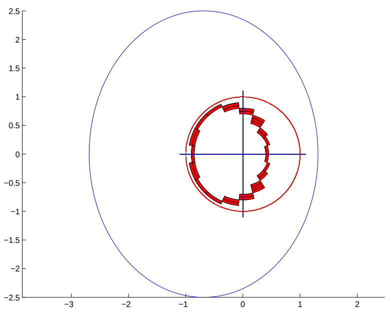

Figure 2. the UC representation of the end-effector of a robot, its qualitative length parameter and orientation angle are set as 19 and 20

2.2 Qualitative Velocity

Definition 2: The qualitative velocity of the end-effector of a robot, qv, consisting of qualitative linear velocity, qv_l, and qualitative angular velocity, qv_θ, is the derivative of the qualitative position of a state such as qualitative length, qp_l, and qualitative orientation, qp_θ. For the qualitative velocity of a robot, we have the following:

$$qv_l = \frac{dqp_l}{dt} \approx \frac{\Delta qp_l}{\Delta t}, \quad qv_\theta = \frac{dqp_\theta}{dt} \approx \frac{\Delta qp_\theta}{\Delta t}$$

The relationship between the certainty values of particular values is characterised by the partial derivative, qv_l and qv_θ are given by the mean value theorem of differentiation. The direction of the velocity can be calculated from the following,

$$[\Delta qv_l] = \left[\frac{\partial qv_l}{\partial qp_l}\right][\Delta qp_l]$$

$$[\Delta qv_\theta] = \left[\frac{\partial qv_\theta}{\partial qp_\theta}\right][\Delta qp_\theta]$$

Further, the qualitative description of general velocity of an end-effector is derived.

$$[\Delta qv] = \oplus [\Delta qv_l] \oplus [\Delta qv_\theta] = \oplus \frac{\partial qv_l}{\partial qp_l}[\Delta qp_l] \oplus \frac{\partial qv_\theta}{\partial qp_\theta}[\Delta qp_\theta] \quad (2)$$

2.3 Qualitative Acceleration

Definition 3: The qualitative acceleration of the end-effector of a robot is the derivative of the velocities of the state, or the double derivative of the state. Firstly we have,

$$qa_l = \frac{dqv_l}{dt} = \frac{d^2qp_l}{d^2t} \approx \frac{\Delta qv_l}{\Delta t} \approx \frac{\Delta qp_l}{\Delta^2 t}$$

$$qa_\theta = \frac{dqv_\theta}{dt} = \frac{d^2qp_\theta}{d^2t} \approx \frac{\Delta qv_\theta}{\Delta t} \approx \frac{\Delta qp_\theta}{\Delta^2 t}$$

Then,

$$[\Delta qa_l] = \frac{\partial qa_l}{\partial qv_l}[\Delta qv_l] = \frac{\partial qa_l}{\partial qp_l}[\Delta qp_l]$$

$$[\Delta qa_\theta] = \frac{\partial qa_\theta}{\partial qv_\theta}[\Delta qv_\theta] = \frac{\partial qa_\theta}{\partial qp_\theta}[\Delta qp_\theta]$$

where $[\Delta qa] = sign(qa_{k+1} - qa_k)$. Finally, the qualitative description of general acceleration is derived,

$$[\Delta qa] = \oplus [\Delta qa]_l \oplus [\Delta qa_\theta] = \oplus \frac{\partial qa_l}{\partial qv_l}[\Delta qv_l] \oplus \frac{\partial qa_\theta}{\partial qv_\theta}[\Delta qv_\theta]$$

$$= \oplus \frac{\partial qa_l}{\partial qp_l}[\Delta qp_l] \oplus \frac{\partial qa_\theta}{\partial qp_\theta}[\Delta qp_\theta] \quad (3)$$

3 Conclusion

In this paper a novel qualitative modelling of kinematic robots has been proposed. First, position and orientation properties of the end-effectors of robots and their link segments are derived. Second analytical formulas of qualitative velocity and qualitative acceleration are derived. The approach provides a potential bridge between traditional robotics and cognitive robotics.

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