# A Qualitative Representation of Trajectory Pairs 

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There has been much research in temporal and spatial reasoning, both quantitative and qualitative, the latter being of particular interest from a cognitive viewpoint [2]. Attempts to combine both spatial and temporal relationships include [1,7,9]. A database approach to specifying spatial relationships that hold between moving objects during a particular interval is [10]. An approach that combines topological relationships between regions in 2D space with temporal relationships between convex intervals is [1].

However, we believe the question remains of how to describe motion adequately within a qualitative calculus. Motion can be divided into change of location, i.e. translation, and change in orientation, i.e. rotation. We focus on the former aspect here. A thorough investigation into mereotopological spatio-temporal continuous change has been conducted in [6], though there has been little work on describing relative motion of disconnected objects. However, it is clear that mobile disconnected objects (animals, vehicles,...) are prevalent in many domains and it would be highly desirable to be able to describe their motion in a qualitative manner. One move in this direction is the extension of qualitative physics to handle relative positions of objects in 2D [11], but this relies on projecting positions to $x$ and $y$ axes and does not provide a calculus with a set of jointly exhaustive and pairwise disjoint (JEPD) relationships. A simple calculus for describing traffic events is [4]. The work presented in this paper can be viewed as a continuation of these strands of previous research; i.e. it is an exploration of trajectories of moving (point like) objects.

If objects do not change their form during the movement and we focus on the representation of spatially disjoint objects, then we can take an arbitrary point (e.g. the centroid) as the spatial location of an object. Therefore in this paper objects are represented simply as points. Moving objects can be partitioned in those having a free trajectory and those with a constrained trajectory [8]. A free trajectory means that there are no significant restrictions on the movement of a point in an nD space, such as an airplane traveling through the sky. A constrained trajectory means that the movement of an object in space is strongly restricted. A 1D representation can provide a useful abstraction for many free trajectory applications; e.g. even though a prey and a predator move in nD , the vital question is whether or not the predator catches the prey (represented by their Eucledian distance apart).

Positional information is determined by the orientation and the distance relation [3]. Based on this and the notion of mode space (in which a space is subdivided in homogenous clusters) [6], the movement or transition between two objects at an instant can be qualitatively represented using three functions:
i. movement of the 1st object wrt the 2nd object's position
ii. movement of the 2nd object wrt the 1st object's position
iii. relative speed of the 1 st object wrt the 2nd object

Since we are interested in a qualitative calculus, we can represent the values of each of these functions by " + ", " 0 " or " - " (cf [13]).

[^0]For the first two functions, we take "-" to mean motion towards the other object, " + " to mean motion away, and " 0 " to mean an absence of motion to/from the other object. In (iii), " $+/ 0 /-$ " mean a greater/same/lower speed respectively. This triple function forms the basis of our Qualitative Trajectory Calculus (QTC).

In order to clarify what we mean by motion towards/away another object, consider Fig. 1. In each case the position of $l$ at time $t$ is indicated with a black dot. The trajectory of $k$ is indicated with an arrow such that at time $t$ it is exactly at the intersection of the circle centered at $l$. In a. we say that $k$ is moving toward $l$ at $t$ $(-)$; in b. that $k$ is moving away $(+)$, and in c. and d., that $k$ is (instantaneously) stationary ( 0 ) with respect to the position of $l$ at $t$.

We can represent a trajectory by a label consisting of 3 characters, each one giving a value for (i)-(iii) above. Thus there are $27\left(3^{3}\right)$ potential trajectory pairs. When only the two movement constraints are considered, there are 9 trajectories - see Fig. 2.

Fig. 2
Fig 1.


Figure 1. Qualitatively different cases of motion of $k$ wrt $l$ at $t$.
Figure 2. Visualization of Qualitative Trajectory Pairs in a conceptual neighborhood diagram. The dots represent the positions of the objects (solid: object can be stationary, open: object cannot be stationary). The lines and crescents represent the potential object movements (if a crescent is used, then the movements start in the dot and ends somewhere on the curved side of the crescent).

There are 6 basic trajectory pairs of which three have an inverse ( -0 inverse to $0-$, -+ inverse to +- , and +0 inverse to $0+$ ) and the other are self-inverse (,--++ , and 00 ).

Freksa [5] introduced the important idea of conceptual neighborhood:'Two relations between pairs of events are conceptual neighbors, if they can be directly transformed into one another by continuously deforming (i.e. shortening, lengthening, moving) the events in a topological sense'. In QTC, two trajectory pairs are conceptual neighbors if they can directly follow each other during a continuous movement. We can analyze this in terms of the continuity of each component of a QTC triple. It is clear that " - " and " 0 " are neighbors and " 0 " and " + " are also neighbors whilst " - " and " + " are not, since a change of value from " + " to " - " has to go through 0 on the real number line if changes are assumed to be continuous. Thus the trajectory pairs "- +" and " 0 +" are
conceptual neighbors since the second characters are identical ( + ) and the first characters are neighbors $(-, 0)$. On the other hand "- +" and "+ +" are not since they can only be linked via the " 0 +" transition. Together with the theory of dominance space [6], this results in the conceptual neighborhood presented in Fig. 2.

To clarify the way in which trajectories are represented within QTC it may be helpful to consider some examples. A particularly interesting case is that of circular motion. Consider the situations depicted in Fig. 3 where two objects are traveling along the same circular path (shown with a thin continuous line). In Fig. 3a $k$ and $l$ are diametrically opposite at time $t$. If $k$ moves anywhere below the dashed line (which is perpendicular to the segment joining $k$ and $l$ ), then it will be moving closer to where $l$ was at time $t$. However just before t , k was moving away from where l was at time t .
Therefore the appropriate qualitative value representing the relationship between $k$ and $l$ is " 0 " at time $t$, "-" just before $t$, and " + " just after $t$. Dual reasoning applies for the movement of $l$ with respect to the position of $k$ at $t$; so we have 00 at $t$. Note that it is irrelevant whether the objects are moving clockwise or anticlockwise. Now assume that both objects from Fig 3b are traveling clockwise. It can be seen that $k$ is moving away from $l$ and $l$ is moving towards $k$, so the description is +- . If the motion were anticlockwise, then the description would be - +.


Figure 3. Circular Trajectories
An important task in qualitative dynamics is to be able to represent the relationship between specified individuals over an interval. We illustrate this task with an example consisting of the evolution of the interaction between a carnivore and its prey. When a carnivore hunts a prey, the mereotopological relationship is typically that of disjointness until the time that the prey is caught. We now describe a hunt, both informally in English, and with annotations in QTC:
(1) A resting lion sees a resting zebra and starts stalking the zebra. Conceptual path of qualitative trajectory pairs (CPT $00,-0$ )
(2) All of a sudden the zebra gets a glimpse of the lion and tries to escape. (CPT - $0,-++,-+0,-+-)$
(3) The lion reacts, and starts following the zebra with a higher velocity. (CPT -+-, -+ $0,-++$ )
(4) After a while the lion gets tired and is not able to run as fast. (CPT -++, -+ $0,-+-$ )
(5) The lion realizes that he will have to do it without food, stops chasing the zebra and takes a rest. (CPT -+ -, 0 +)
(6) After a while, the zebra is certain that he has got rid of the lion, stops running and continues with grazing. (CPT $0+, 00$ )
The composition of these qualitative trajectory pairs can be visualized using the conceptual neighborhood diagram (see Fig. 4).

In many environments, moving objects such as cars tend to follow predefined spatial paths namely roads, highways, etc. In [12] we outline how the calculus might be specialized to this case.

We believe that the proposed approach may be a useful starting point for analyzing the complex interaction between moving objects. Although the 1D case can be seen as a solid basis, extensions have to be made to 2D and 3D. Another important issue for future research is to investigate the composition tables and the complexity of reasoning in the calculus. Our initial investigations have shown that unless restricted to a pure 1D domain, the composition table is rather weak; however we are also working on extending the relational calculus with orientation knowledge (e.g. 14] which will allow a more useful composition table).


Figure 4. Carnivore-prey example. Note that this figure extends the conceptual neighbourhood diagram displayed in Fig 2 by representing also the third element of a QTC-triple.
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