Defining Classes of Influences for the Acquisition of Probability Constraints for Bayesian Networks

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Abstract. The task of eliciting all probabilities required for a Bayesian network can be supported by first acquiring qualitative constraints on the numerical quantities to be obtained. Building upon the concept of qualitative influence, we analyse such constraints and define a small number of influence classes. Based upon these classes, we present a method for efficiently acquiring the qualitative constraints that should be satisfied by the network's probabilities.

1 INTRODUCTION

Bayesian networks for real-life applications are often constructed with the help of domain experts. A Bayesian network is a concise representation of a probability distribution that consists of a graphical part, encoding the relevant variables of the domain along with their probabilistic interrelationships, and a numerical part, encoding the conditional probabilities that represent the strengths of these relationships. Experience shows that, although it may require considerable effort, configuring the graphical part of a network is quite practicable. In fact, well-known knowledge-engineering techniques for designing domain models can to some extent be employed for this task [1]. Obtaining the probabilities for the numerical part of the network, however, is generally considered a far harder task, especially if these probabilities have to be assessed by domain experts [2].

Recently, a methodology for building Bayesian networks has been introduced in which the specification of qualitative influences has been proposed as an intermediate step in the construction of a network's numerical part [3]. These influences then are taken as constraints on the probabilities to be obtained. In this paper, we further elaborate on this idea. We study the different orderings of the probabilities to be specified for a variable and its causes, and show that these orderings give rise to different combinations of influences. We then define a small number of classes of influences.

Now, upon building a Bayesian network, a combination of qualitative influences has to be specified for each variable and its causes. We present a method for acquiring knowledge for this purpose from domain experts. The method builds upon the idea of establishing, for each variable, the appropriate influence class. From just a partial ordering of the probabilities involved, some of the classes and the associated combination of influences can be identified uniquely; for the other classes, additional knowledge has to be acquired before the combination of influences is fully specified. The classes thus serve to guide the knowledge engineer in her acquisition efforts. Preliminary results of the use of our method in a real-life application domain have demonstrated its practicability.

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2 PRELIMINARIES

A Bayesian network is a model of a joint probability distribution, consisting of a graphical part and an associated numerical part. The graphical part is an acyclic digraph, in which each node A represents a statistical variable. For ease of exposition, we assume all variables to be binary, taking one of the values *true* and *false*; we use a to denote $A =$ *true* and $\bar{a}$ to denote $A =$ *false*. We further assume that a variable's values are ordered, where *true* $>$ *false*. The arcs in the digraph model the probabilistic influences between the represented variables. Informally speaking, an arc $B \rightarrow C$ between the nodes B and C indicates a direct influence between the associated variables; B then is referred to as the cause of the effect C. The variable C with all its possible causes constitute a causal mechanism, an example of which is shown in Figure 1. Associated with the digraph of the network are numerical quantities. With each variable C is associated a set of conditional probability distributions $\Pr(C \mid \pi(C))$; each of these distributions describes the joint effect of a specific combination of values for the causes $\pi(C)$ of C, on the probabilities of C's values.

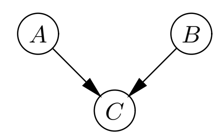

Figure 1. A basic causal mechanism

A qualitative influence between two variables expresses how observing a value for the one variable affects the probabilities for the other variable [4]. For example, a positive qualitative influence of a variable B on a variable C along an arc $B \rightarrow C$ expresses that observing a higher value for B makes the higher value of C more likely, regardless of any other direct influences on C, that is,

$$\Pr(c \mid bx) - \Pr(c \mid \bar{b}x) \geq 0$$

for any combination of values x for the set $\pi(C) \setminus \{B\}$. The influence is denoted $S^{+}(B, C)$, where the '$+$' is termed the sign of the influence. A negative qualitative influence, denoted by S^{-}, and a zero qualitative influence, denoted by S^{0}, are defined analogously, replacing the inequality $\geq$ in the formula above by $\leq$ and $=$, respectively. If the influence of B on C is positive for one combination of values x and negative for another combination, then the influence is called ambiguous and has associated the sign '?'.

3 CLASSES OF INFLUENCES

Different combinations of influences may hold for a causal mechanism. For reasons of space, we focus in this paper on mechanisms with two causes only, as exemplified in Figure 1. For the effect C of the basic causal mechanism, the four conditional probabilities $\Pr(c \mid ab)$, $\Pr(c \mid \bar{a}b)$, $\Pr(c \mid a\bar{b})$ and $\Pr(c \mid \bar{a}\bar{b})$ are required. For these probabilities, there are 24 possible orderings, each of which gives rise to a specific combination of influences. We now define four different classes of combinations of influences.

Class *I* is based upon the eight orderings of the probabilities under study in which the presence of the common effect is less likely in the situations in which just one of the two causes is present than in the two situations where both causes are either present or absent:

$$\begin{array}{lcl}\{\Pr(c \mid ab), \Pr(c \mid \bar{a}\bar{b})\} & \geq & \{\Pr(c \mid \bar{a}b), \Pr(c \mid a\bar{b})\} \\ \{\Pr(c \mid ab), \Pr(c \mid \bar{a}\bar{b})\} & \leq & \{\Pr(c \mid \bar{a}b), \Pr(c \mid a\bar{b})\}\end{array}$$

The notation $\{p, q\}$ in the orderings above is used to indicate that p and q are ordered arbitrarily. For the four orderings that involve the inequality $\geq$, we find that $S^?(A, C)$ and $S^?(B, C)$. Since $\Pr(c \mid ab) \geq \Pr(c \mid \bar{a}b)$ and yet $\Pr(c \mid a\bar{b}) \leq \Pr(c \mid \bar{a}\bar{b})$, we have that the sign of the influence of A on C depends on the value of the other cause of C: with $B = $ *true*, the influence of A on C is positive; with $B = $ *false*, it is negative. The overall influence of A on C, therefore, is ambiguous. A similar observation holds for the influence of B on C. For the four orderings that involve the inequality $\leq$, we equally find that $S^?(A, C)$ and $S^?(B, C)$. Class *I* thus captures the combination of two ambiguous influences.

Class *II* is based upon the four orderings of the probabilities under study in which the presence of the effect is more likely in the situations in which just one of the causes is present than in the situation in which both causes are absent, yet less likely than in the situation in which both causes are present:

$$\begin{array}{lclcl}\Pr(c \mid ab) & \geq & \{\Pr(c \mid \bar{a}b), \Pr(c \mid a\bar{b})\} & \geq & \Pr(c \mid \bar{a}\bar{b}) \\ \Pr(c \mid ab) & \leq & \{\Pr(c \mid \bar{a}b), \Pr(c \mid a\bar{b})\} & \leq & \Pr(c \mid \bar{a}\bar{b})\end{array}$$

For the two orderings that involve the inequality $\geq$, we find that $S^+(A, C)$ and $S^+(B, C)$. Since $\Pr(c \mid ab) \geq \Pr(c \mid \bar{a}b)$ and $\Pr(c \mid a\bar{b}) \geq \Pr(c \mid \bar{a}\bar{b})$, we have that the influence of A on C is positive regardless of the value of B. The influence of A on C, therefore, is positive. A similar observation holds for the influence of B on C. For the two orderings that involve the inequality $\leq$, we find that $S^-(A, C)$ and $S^-(B, C)$. Class *II* thus captures the combination of two influences with the same unambiguous sign.

Class *III* is based upon the four orderings of the probabilities under study in which the presence of the common effect is more likely in the two situations in which both causes are either present or absent than in the situation in which just the one cause is present, yet less likely than in the situation in which just the other cause is present:

$$\begin{array}{lclcl}\Pr(c \mid \bar{a}b) & \geq & \{\Pr(c \mid ab), \Pr(c \mid \bar{a}\bar{b})\} & \geq & \Pr(c \mid a\bar{b}) \\ \Pr(c \mid \bar{a}b) & \leq & \{\Pr(c \mid ab), \Pr(c \mid \bar{a}\bar{b})\} & \leq & \Pr(c \mid a\bar{b})\end{array}$$

For the two orderings that involve the inequality $\geq$, we find that $S^-(A, C)$ and $S^+(B, C)$. For the other two orderings, we find that $S^+(A, C)$ and $S^-(B, C)$. Class *III* thus captures the combination of two influences with opposite unambiguous signs.

Class *IV*, to conclude, is based upon the eight orderings in which the probabilities of the effect c given the same observations for the two possible causes are interleaved in the ordering with the probabilities of c given opposite observations for the two causes. An example of such an ordering is:

$$\Pr(c \mid ab) \quad \geq \quad \Pr(c \mid \bar{a}b) \quad \geq \quad \Pr(c \mid \bar{a}\bar{b}) \quad \geq \quad \Pr(c \mid a\bar{b})$$

For all eight orderings involved, we find that $S^?(V, C)$ for some $V \in \{A, B\}$ and that $S^\delta(W, C)$ for some $\delta \in \{+, -, 0\}$ and $W \in \{A, B\}$ with $W \neq V$. In the ordering mentioned above, for example, we have that $\Pr(c \mid ab) \geq \Pr(c \mid \bar{a}b)$ and yet $\Pr(c \mid a\bar{b}) \leq \Pr(c \mid \bar{a}\bar{b})$. We conclude that the influence of A on C is ambiguous. We further observe that $\Pr(c \mid ab) \geq \Pr(c \mid a\bar{b})$ and $\Pr(c \mid \bar{a}b) \geq \Pr(c \mid \bar{a}\bar{b})$. The influence of B on C is therefore positive. Class *IV* thus captures an ambiguous influence of one of the causes and an unambiguous influence of the other cause.

4 ACQUIRING INFLUENCES FROM EXPERTS

To specify probability constraints for a Bayesian network, combinations of qualitative influences are to be established for all causal mechanisms involved. We designed a method for acquiring the relevant knowledge to this end, that builds upon the four classes of influences defined above. Our method begins by establishing the situations in which the presence of the effect is the most likely and the least likely. If these situations indicate that the combination of influences for the mechanism under study belongs to one of the classes *II* and *III*, then the partial ordering of probabilities obtained serves to uniquely determine the signs of the influences involved: these can simply be looked up and do not have to be acquired from the domain expert. If the combination of influences under study is found to belong to one of the classes *I* and *IV*, then a total ordering of the probabilities involved is obtained; once the total ordering is acquired, the signs of the influences can again be looked up. Note that by thus building upon the orderings of probabilities, we circumvent any misinterpretation of the concept of influence by the experts.

5 CONCLUDING OBSERVATIONS

We designed a method for acquiring qualitative influences to be used as constraints on the probabilities for a Bayesian network. For this purpose, we studied the different orderings of the probabilities to be specified for a causal mechanism and, based upon these orderings, defined a small number of classes of influences. Our method now builds upon the observation that from just a partial ordering of the probabilities involved, the influence class of a causal mechanism can often be derived, thereby uniquely defining the signs of the influences involved. We conducted an initial study of the use of (an extended version of) our method with an intensive-care neonatologist to acquire probability constraints for a small part of a real-life Bayesian network under construction [5]. The preliminary results indicate that the method takes relatively little effort on the part of the expert.

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